PERSPECTIVE

AN ELEMENTARY TEXT BOOK

BY

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A DRAFTSMEN'S HAND BOOK"

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PREFACE

THIS book is intended principally for the struggling student who is endeavoring to better himself by home study and who can get but little assistance, if any, either personal or from books, the latter usually being too difficult for him to read. It is intended to give him a beginning so that he may be able to solve the ordinary problems of every-day practice, and, as stated in the introduction, to qualify him for the reading and study of the more profound books on the subject. The general discussions may also be read with profit by laymen who wish merely a general knowledge of the science. The text is the result of many years' experience in making perspective drawings in an architect's office and in teaching the science to beginners. The manuscript was tried, with marked success, by several novices who had not before known anything about the subject, and who were enabled after a careful study of the book to lay out an ordinary perspective accurately and without help.

If, in some measure, this book succeeds in doing this for others, if it lightens the task and straightens the road for but a few, the satisfaction of the author will far exceed that of those helped.

The student is assumed to have some knowledge of plane geometry, but this, though desirable, is not absolutely necessary. It is, of course, essential that he have some familiarity with architectural or mechanical drawing.

B. J. L.

KANSAS CITY, MISSOURI,
December tenth, nineteen twelve.

PREFACE TO THIRD EDITION

THE encouraging success of the two previous editions of this little book has made it seem desirable to issue a third.

The new material in the second edition was incorporated principally at the suggestion of the late Professor Frank Dempster Sherman of Columbia University. A short description of the use of circumscribed octagons as an aid in drawing perspective circles has been included in this edition, in view of a later suggestion by Professor Sherman for increasing the usefulness of the text.

A chapter on the history of perspective drawing has also been added in this issue. The data for this chapter, short as it is, was gathered from scattered sources and after the examination of many old volumes. Usually this historical material would be placed first as an introduction, but in keeping with the original plan of the book, which was based on the natural laboratory method of teaching, it has been placed last, on the assumption that the student would, after an acquaintance with the theory and principles of perspective, be much more interested in its history than before.

The popularity of this little book has been the source of much gratification to the author, who hopes that its continued usefulness will more than justify this republication.

B. J. L.

NEW YORK CITY,
May, nineteen twenty-one

CONTENTS

CONTENTS

CONTENTS

I

INTRODUCTORY

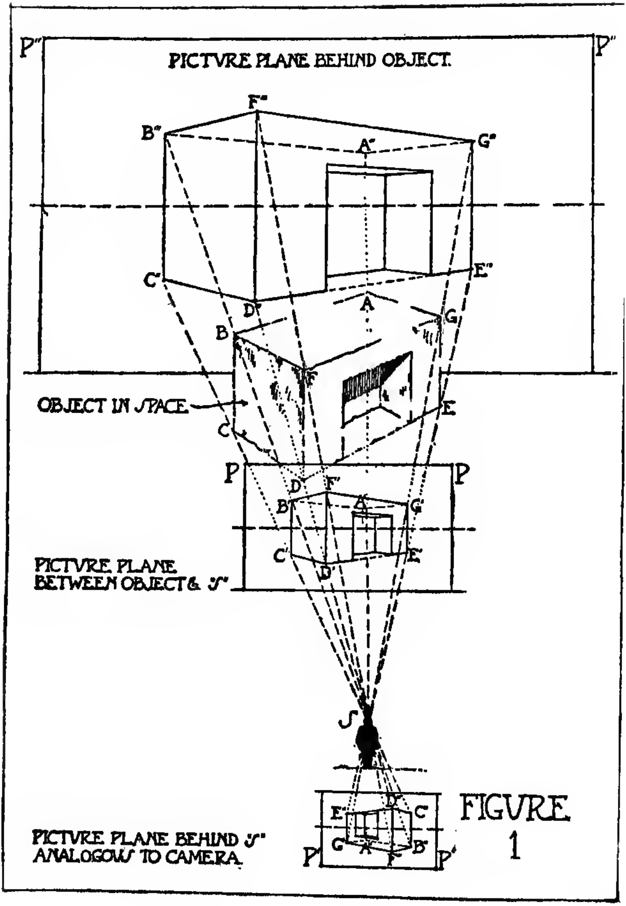
THERE are so many books on Perspective that an apology is necessary for inflicting another. The books, however, with which the writer is familiar, although excellent texts, usually do not begin at the very beginning, and the inexperienced student cannot read them understandingly. Not so many years ago, the writer was a student-beginner himself, and he still remembers quite vividly the yearning for knowledge at that time. This yearning, in so far as concerned Perspective, was not for the knowledge of the science and theory of the subject, but rather for the ability to make a perspective drawing. So in this little book, after a preliminary talk and a few observations on Perspective and its phenomena, we are going to make, you and I, a perspective drawing; we shall work together step by step and line by line. Then you will make another perspective drawing, using the same methods that we used together, but without the explicit instructions. Having done this, we hope that your interest will have been sufficiently aroused so that you will want to know why you did what you did, and then, again step by step and line by line, you will be told why. You will be told enough only, so that by deductive thinking you may be able to work out the ordinary problems encountered, or, better yet, so that you may be able to read some of the admirable existing texts on the subject. It is not the intention to encroach on the field of such excellent books as Ware's "Modern Perspective" or Longfellow's "Applied Perspective."

This, in brief, is the plan of this little book. It follows what may be called the "laboratory method" of modern teaching. Some of us have found that the way to learn, and the way we do learn, a new language is by speaking and reading it first, and then studying its grammar and rhetoric. So we shall do with Perspective. We shall learn first to make a perspective drawing —indeed, we shall make one or two—then we shall study its grammar and rhetoric.

II

PRELIMINARY

EVERYTHING we see, we see in perspective. Every image in the eye is in perspective, a perspective projection; every photograph is in perspective and a perspective projection. If the student should erect a glass screen between himself and an object such as a building and run imaginary lines from his eye to various points on the object, say the corners, where these lines would pierce the glass screen would be the perspective projections of the corresponding points on the object. If we now connect these projected points on the screen by lines, we shall have a picture or perspective projection of the object on the screen. We must always remember this: the perspective projection of an object is a picture, an image of it on an assumed plane. It is with the construction of this image or projection, *without actually making the projections* of the various defining points, that the science of Perspective has to deal. Let us get back for a moment to the glass screen. In the parlance of Perspective this screen is called the picture plane; the position of the observer's eye, the


PICTVRE PLANE BEHIND OBJECT.
P"
P"
F"
B"
A"
G"
C"
E"
D"
A
G
B
E
C
OBJECT IN SPACE
P
P
D'
F'
B'
A'
G'
C'
E'
D'
PICTVRE PLANE
BETWEEN OBJECT & "S"
S
E'
D'
C'
G'
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P'
PICTVRE PLANE BEHIND "S"
ANALOGOUS TO CAMERA.

FIGURE
1

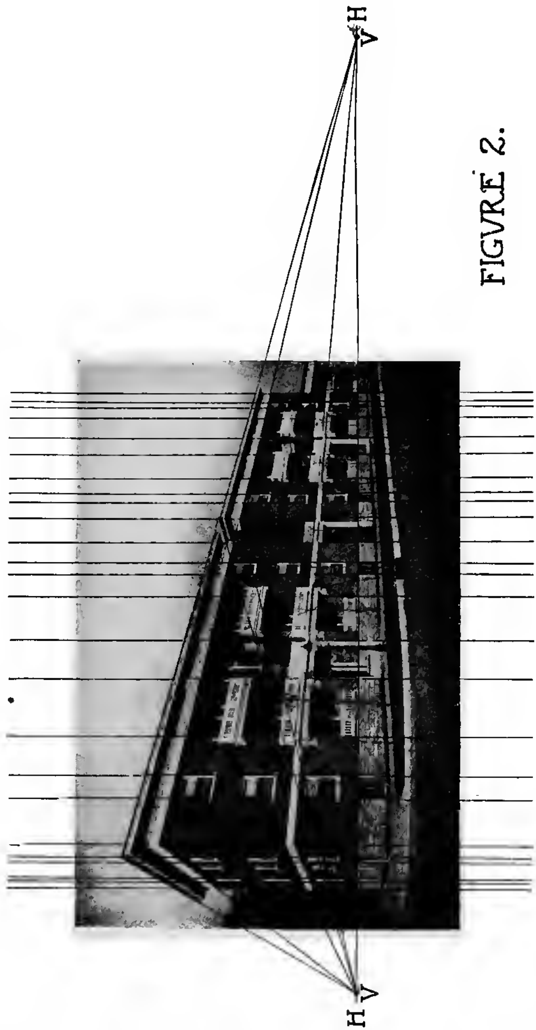
point of station. Although not necessarily so always, the picture plane is usually assumed to be vertical. We are now ready for an explicit definition of a perspective (Figure 1).

A perspective of an object is the projection of the defining lines and points (A, B, C . . .) of that object on a plane (PP) called the picture plane. This projection is formed by drawing lines from the defining points (A, B, C . . .) on the object to a point (S) called the point of station. The intersections (A', B', C' . . .) of these lines (SA, SB, SC . . .) with the picture plane (PP) define the projection or so-called perspective (A', B', C').

A photograph is a true perspective In Figure 1, if we continue the lines SA, SB, SC, etc., through the point of station, S, to a plane parallel to the picture plane, PP, as $P'P'$, and intersecting $P'P'$ in A'', B'', C'' . . ., the inverted image (A'', B'', C'' . . .) thus formed is similar to the projection A', B', C' . . . on PP, and is analogous to the image formed on the ground glass or focussing screen of the camera. The ground glass of the camera corresponds to $P'P'$, and the lens of the camera to S, the point of station.

Having shown that a photograph is a true perspective projection, we shall now study a photograph and note some of the peculiar phenomena of perspective (Figure 2). We see that vertical lines on the object or building remain vertical in the photograph. This we must remember. As long as the picture plane is vertical—and we shall not consider cases where it is not—verticals in the object will remain vertical in the picture. We see also that horizontal lines in the object converge in the photograph. We should note that when these horizontal lines are parallel, such as those lying in the same or parallel sides of the building, they converge in the same point, as *V* and *V′*. These points, *V* and *V′*, are called the vanishing points for the lines which converge in then. Note that the line *HH* drawn through *V* and *V′* is horizontal. This line is called the horizon of the picture. It is the neutral line of the perspective and is always on the level of the point of station or observer's eye. All horizontal lines below the level of the horizon incline upward toward it, and all horizontal lines above the level of the horizon incline downward toward it. The vanishing points of all horizontal lines lie in the horizon. Finally, we should note that equal distances and dimensions on the object decrease in magnitude on the picture as they recede. This diminution is due either to increase of distance from the eye, to the obliquity of view, or foreshortening, or to both.

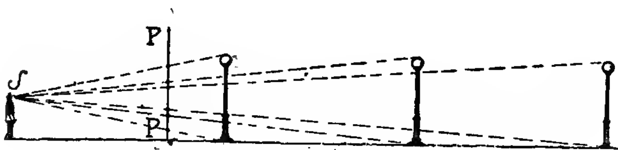
In later chapters of the book all these phenomena will be taken up, discussed and explained, but a few words in explanation of why parallel lines converge and vanish in the same point and why parallel horizontal lines converge and vanish in the same point in the horizon may not be amiss now.



FIGVRE 2.

To face p. 6—Lubschez "Perspective"

The size of an image on the retina of the eye or the picture plane of a perspective depends on the size of the angle subtended by the object seen or pictured; that is, if lines be drawn from the extremities of a dimension of the object to the point of station or eye, then the size of the image or projection of that dimension depends on the angle between these lines. The farther away the object, the smaller the angle and hence the smaller the image. This is obvious from Figure 3:



FIGVRE 3.

When this distance of the object becomes infinite, the angle subtended becomes infinitesimal and the image a point. Practically speaking, of course, the image becomes a point when the subtended angle becomes so small that it cannot be recognized as such by the eye. This happens at a comparatively great distance but a finite one. Now let us imagine a long straight railroad track in the direction of which we are looking. Each successive tie between tracks subtends a lesser angle at the eye and hence makes a smaller image. At a great distance this image becomes practically a point and the two lines of track seem to meet. This point is the so-called vanishing point for the lines of the track. Likewise, it may be seen that in any system of parallel lines—not parallel to the picture plane —the lines will converge to a common vanishing point; this being, for the same reason, the decrease in the apparent distance between them. Assuming for practical purposes that as much of the earth's surface as we can see at one time is a plane, then any line lying in this surface or plane and at an angle with the picture plane must vanish in the horizon or apparent end of this plane, and any line parallel to the first line, and therefore a horizontal line, must vanish in the same point on the horizon. This should make it clear that the vanishing points of horizontal lines must lie in the horizon, but the hypothesis will be more easily understood later when the subject of parallel planes and their vanishing lines will be taken up.

We should also remember that the position of the picture plane is entirely arbitrary (Figure 1). It may be placed between the object and the point of station, the logical position for it; it may be behind the point of station, as in a camera when the projected image is inverted; it may be behind the object when the projected image is larger than the object; finally, it may pass through the object, as is sometimes the case in architectural perspective where the picture plane is assumed to pass through some convenient corner of the building and some projection such as a porch, stoop or cornice lies wholly or partly in front of it. The principal thing to remember is that wherever the picture plane may be, we have but to continue our projecting lines from the point of station to it to form the perspective projection. This will also be discussed and explained more fully later.

We are now, in the next chapter, going to solve a simple but typical problem in perspective; or, in other words, we are going to make a perspective drawing.

III

MAKING A DRAWING IN PERSPECTIVE

LET us consider for a moment what we do in making elevations to fit a certain floor plan or horizontal section. What we do, though we may do it indirectly, is to project lines vertically from the defining points of the plan, then draw the horizontal divisions; in this way the elevations are drawn or constructed. The making of a perspective is analogous. We first draw our plan in perspective; we draw it as if it were laid out on the ground and we were looking at it from our point of station. We then project lines vertically from the defining points of the perspective plan, and next we draw the horizontal and other lines in perspective connecting these verticals; thus we get our elevations in perspective. The combination of contiguous perspective elevations or their projections makes the complete perspective. This will be quite clear from Figures 4 and 5. (See also Figures 6 and 8.) Of course, we may not do all this directly and explicitly, especially when we become proficient; but, in fact, that is what the process of constructing a perspective projection amounts to.

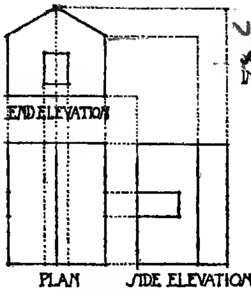


FIGURE 4

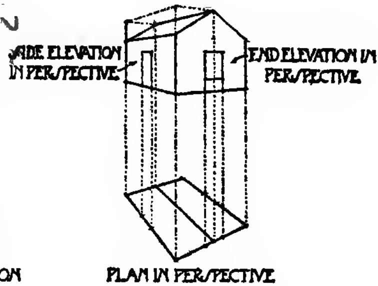


FIGURE 5

Figure 6 shows a plan and two elevations of a simple monument of which we shall make a perspective drawing. The monument is shown on a platform three steps high. The student would do well to copy this figure at the scale of half an inch to the foot, first, to familiarize himself with the lines of the monument, and second, to have the drawings for reference in making the perspective, which is to be at the same scale of half an inch to the foot.

Next we should make the diagram shown in Figure 7. We first make an outline plan of the object such as *A, C, B,* at any convenient scale, say half that of the perspective, or one quarter of an inch to the foot. Of course, the simplest thing to do would be to use the same scale for this as for the perspective, but this would make the diagram too large. We may just as well make it smaller and transfer the points found on it at the proper scale. Let us now locate on this diagram the point *S*, which marks the point of station. In this case it is thirty feet from *C*, the nearest corner of the object or platform of monument, and on the line *SC*, which makes an angle of 120° with *BC*. We must remember that the location of *S* is entirely arbitrary and left to our choice. Its proper location determines the pictorial value of our perspective, and we must depend on judgment, experience and observation of nature to locate it properly and effectively.

We now draw through *C* a line *PP*, perpendicular to *SC*. This line, *PP*, is the trace or plan of our picture plane.

If we now draw through *S* a line parallel to *AC*, where this line intersects *PP* at *V* is the vanishing point of all lines parallel to *AC*.

Likewise a line through *S* parallel to *CB* will give us, at *V'*, the vanishing point for all lines parallel to *CB*. This we should note very carefully: we may find the vanishing point for any line or its parallel by drawing a line parallel to it through the point of station. The intersection of this line with *PP* will be the required vanishing point.

We next find *M* and *M'*, the measuring points; these points are used to foreshorten horizontal distances on the perspective plan. In other words, as will be shown in Figure 8, *M* is used to find the perspective distance of a point from C whose actual distance from C is known. Likewise, M' is used to find distances on CA. This will be taken up more fully later.

To find these measuring points, M and M', we must but remember that the measuring point for any line is the same distance on the horizon from the vanishing point for that line as the vanishing point is from the point of station, or as, in Figure 7, $V'M$ equals $V'S$ and VM' equals VS. To find M, then, we take V' as center and $V'S$ as radius, and draw the arc SM; $V'M$ then equals $V'S$. To find M' we take V as center and VS as radius, and draw the arc SM'; VM' then equals VS.

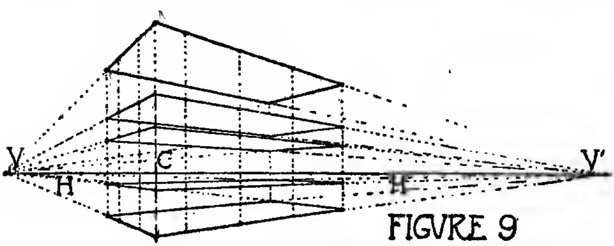
We have now found all the points we shall need to use in the finding of the various perspective projections. We have the vanishing points, V and V', for the two systems of lines we have to deal with (a system of lines in perspective is a group of lines parallel to each other), and the two measuring points, M and M'. We also have C, which is called the center, and S, the point of station. The line CS from the point of station to the center and perpendicular to the picture plane is called the axis. We now turn to Figure 8, the perspective and its plan.

First, in Figure 8 we draw a horizontal line, HH. This is the horizon of our picture. On HH we lay out the points V, V', M, M' and C, as found on the diagram, Figure 7, but at the scale of our perspective.

Through *C* we now draw a vertical line. This vertical line is the line of measures, and on it will be laid off all vertical distances or heights.

On the line of measures, below *C*, which is the projection of the point of station on the horizon, we must lay off the height of the point of station above the ground. The normal height being 5 feet 6 inches, we shall so make it here; *CG* therefore scales 5 feet 6 inches, and *G* represents the ground line at the nearest corner of the large platform.

On the line of measures, a convenient distance below *G*, say four inches actual distance, we place *C′*. This is the corner of our perspective plan and corresponds to *C* of Figures 6 and 7. The location of this point is entirely arbitrary, and *G* should be the logical location for it, being at the level of the ground surface; but to avoid a confusing network of lines we take *C′* low enough to separate the perspective plan from the picture. We might have taken *C′* above the picture and projected down from the perspective plan, which would then be above the picture. *C′* may, in fact, be anywhere on the line of measures



FIGVRE 9

without changing the vertical projections of the perspective plan as shown in Figure 9.

Through C' we draw lines to V and V', and also a horizontal line $P'P'$. On this line $P'P'$ we lay off, to the scale of our perspective, on each side of C', the actual dimensions of the plan in Figure 6, just as if the lines AC and CB of this plan were turned at C until they formed a straight angle of 180°. In other words, in Figure 8, $A'C'$ scales 21 feet 6 inches, the length of AC in Figure 6, and $C'B'$ scales 19 feet, the length of CB in Figure 6. The intermediate points, a', b', c' . . . and p', q', r' . . . likewise correspond to the distances on the plan (Figure 6).

To get the perspective length of $C'B'$ on $C'V'$, or to locate the point B, the perspective of B', we draw a line from B' to M; where this line intersects $C'V'$ is B. Likewise, by drawing a line from A' to M' where it intersects $C'V$ we get A.

The points M and M' are each really, as we shall prove later, the vanishing points of parallel lines which make equal angles with CB and CB' and CA and CA' of Figure 7. In other words, the triangle $C'BB'$ of Figure 8 is the perspective of the triangle CBB' of Figure 7, whose angles, B and B', are equal. The angles, B and B', of Figure 8 are therefore the perspectives of equal angles. Since this is true, the triangle $C'BB'$, Figure 8, is the perspective of an isosceles triangle, and therefore the lines $C'B$ and

$C'B'$ are the perspectives of equal lines. Likewise, AC' and $A'C'$ are the perspectives of equal lines. The proof of the construction for M and M' will be given later.

We take actual scaled distances on $P'P'$ and the line of measures because both these lines really lie in the picture plane, $P'P'$ being the perspective of PP, Figure 7; and we must always remember that a line which lies in the picture plane does not change its magnitude in perspective, because if it lies in the picture plane it is obviously its own projection in that plane.

As we found B and B' in Figure 8, so can we find all the points defining horizontal dimensions, such as a, b, c . . . and p, q, r . . . We have but to locate a', b', c' . . . and p', q', r' . . . on $P'P'$ at actual scaled distances either side of C', as per our plan, then draw lines to M and M' respectively and find their perspectives on $C'B$ and $C'A$ at the intersections a, b, c . . . and p, q, r . . .

From the points a, b, c . . . B and p, q, r . . . A we draw lines to the vanishing points V and V' respectively. These lines and their intersections will give us the lines and points of the perspective plan, or the plan of Figure 6 in perspective.

Before we leave the perspective plan, we must recall the following: a line drawn through a point on $P'P'$, a given distance from C', to M' will cut off the same distance from C' in perspective on $C'A$ or its continuation; a line drawn through a point on $P'P'$, a given distance from C', to M will cut off the same distance from C' in perspective on $C'B$ or its continuation. To illustrate:

In Figure 8,

$C'r$ =perspective of $C'r'$
$C'c$ =perspective of $C'c'$
$C'a''$=perspective of $C'a'$
$C'q''$=perspective of $C'q'$

It is interesting to note that a'' and q'' lie in front of the picture plane. The finding of these points on the continuations of $C'A$ and $C'B$ then gives us a method of finding the perspectives of points when they lie in front of the picture plane.

The perspective plan is now complete, and we must be sure that we understand how we drew every line before we proceed. Of the picture in Figure 8 we have already drawn the horizon, *HH;* we have located on *HH* the points *V, V′, M, M′* and *C;* we have drawn through *C* the line of measures and have located on it the point *G*, 5 feet 6 inches below *C* and the horizon. The numbered points referred to below correspond to homologous points on the elevations (Figure 6).

Let us now project up from the perspective plan points *a, g, B* and *A*, the verticals $2\text{–}a''$, $3\text{–}g''$, $4\text{–}B''$ and $16\text{–}17\text{–}A''$.

On the line of measures place point 1, eighteen inches above *G*. All points on the line of measures to be laid off at the adopted scale, of course.

From G and 1 draw lines to V'. The intersections of these lines with the perpendiculars locate the points 2, a'', 3, g'', 4, B''.

Locate on line of measures points 7 and 8, six and twelve inches respectively above G.

From 7 and 8 draw lines to V', intersecting 3–g'' at 9 and 10.

From 2 and 3 draw lines to V.

Locate 5 over 5′ of plan.

From 5 draw line to V', intersecting 3–6 at 6. 6 should be over 6′ of plan, a test for accuracy.

Draw verticals 6–15, 13–14, and 11–12 over corresponding points of plan.

From g'', 10 and 9 draw lines to V, intersecting these verticals at 12, 11, 14, 13, 15.

From 12, 11, 14, 13, 15 draw lines to V', which define the edges of steps. We have now completed the steps and bulkheads. From 1 and 7 draw lines to V, intersecting the vertical 16–A'' at 16 and 17.

From 16 draw line to V', and from 4 draw line to V.

Fill in curve of terrace, r''–G, free-hand. This completes the steps and platform upon which the monument stands.

On the line 1–2–3–4 locate 18′ over b of plan, and draw vertical 18′–19′.

On the line of measures locate 19″ six inches above 1, and from 19″ draw line to V', intersecting 18′–19′ at 19′.

On line 19″–19′ locate 24′ over *c* of plan, and draw vertical 24′–25′.

On the line of measures place 25″ twelve inches above 19″, and through 25″ draw line to *V*′, intersecting 24′–25′ at 25′.

Locate 30″ on line of measures six inches above 25″, and through it draw line to *V*′.

On this line place 30′ above *d* of plan and draw vertical 30′–44′.

Locate 44″ on line of measures 9 feet 3 inches above 30″, and draw line through it to *V*′.

On this line, which intersects 30′–44′ at 44′, place 50′ over *e* of plan and draw vertical 50′–47′.

On line of measures place 47″ nine inches above 44″ and draw line through it to *V*′, intersecting 50′–47′ at 47′.

Go back to 25′ and through it draw a line to *V*′.

On this line place 33′ above *f* of plan and draw vertical 33′–35′.

On line of measures place 35″ six feet above 19″.

Draw line through 35″ to *V*′, intersecting 33′–35′ at 35′.

Draw verticals 18–19 and 22–23 over corresponding points of plan.

Draw lines from 18′ and 19′ to *V*, intersecting these verticals at 18, 19, 22, and 23.

Draw vertical 20–21 over corresponding point of plan.

From 18 and 19 draw lines to *V*′, cutting 20–21 at 20 and 21.

From 21 draw line to V, and from 23 draw line to V'.

Draw verticals 24–25 and 28–29 over corresponding points of plan.

Through 24′ and 25′ draw lines to V, cutting these verticals at 24, 25, 28 and 29.

Through 24 and 25 draw lines to V'.

Draw vertical 26–27 over corresponding point of plan, cutting these lines in 26 and 27.

Draw verticals 30–44, 31–45 and 32–46 over corresponding points of plan.

Through 30′ and 44′ draw lines to V, cutting 30–44 and 32–46 in 30, 44, 32 and 46.

Through 30 and 44 draw lines to V', cutting 31–45 in 31 and 45.

Through 47′ draw line to V.

Draw verticals down from this line at 47 and 49 over corresponding points of plan.

From 47 draw line to V'.

Draw vertical down from this line at 48 over corresponding point of plan.

Draw miter lines of bevel base, 30–25, 31–27 and 32–29.

On line 25–27 locate 33 and 34, and on line 30–31 locate 38, over corresponding points of plan.

Through 33, 34, and 38 draw verticals 33–35, 34–36, 38–37.

From 35′ draw line to V, cutting 33–35 and 38–37 at 35 and 37.

From 35 draw line to V', cutting 34–36 at 36.

Draw bevel line 33–38.

Through 25″ on line of measures draw line to *V*.

On this line place 25‴ over *w* of plan and draw vertical 25‴–51′.

Place 51 on line of measures, two and one-half feet above 19″.

Through 51 draw line to *V*, intersecting 25‴–51′ at 51′.

On line 25–29 place 39 and 40, and on line 30–32 place 52, over corresponding points of plan.

At 39, 40 and 52 draw verticals.

Through 51′ draw line to *V′*, cutting 39–41 and 52–43 at 41 and 43. Through 41 draw line to *V*, cutting 40–42 at 42.

Draw lines from 43 to *V* and from 42 to *V′*.

Draw bevel line 39–52.

This completes the perspective.

This perspective illustrates in a striking way what was said before, that a perspective is the combination of contiguous perspective elevations or their projections. This is readily seen in Figure 8. Of course, in practice it is not necessary to complete the perspective elevations; a few defining points only are necessary, and as you proceed and learn to solve perspective problems more readily, many short-cuts will occur to you. You must remember that the varied application of a few primary principles is all that is necessary to solve most problems in perspective. It is the variety and readiness of application rather than variety of principles that you must master. The simple perspective you have just completed involves nearly all these necessary primary principles; therefore the foregoing must be thoroughly understood in every step. Accuracy in drawing is also most essential. Usually it takes several projections to find a point in perspective; a small error in the first place may be magnified into a large one by the time the final projection is made, and a very small discrepancy will often mar the appearance of a perspective picture.

After the drawing like Figure 8 has been made, carefully following the instructions, it should be redrawn from memory, using Figure 6, the plans and elevations only. This is very important. When this has been done readily and correctly you should be ready for the next problem, which is stated in the next chapter.

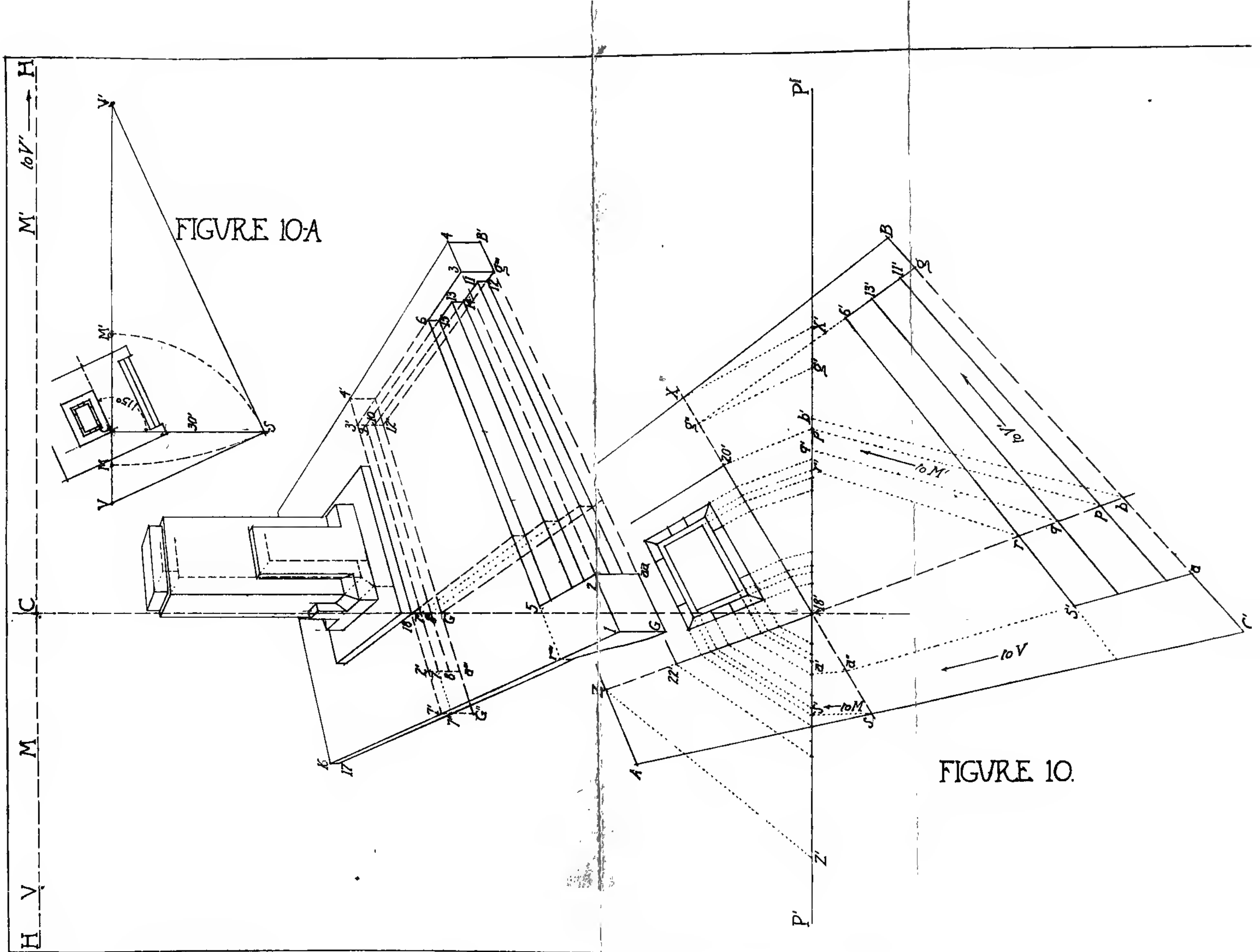


FIGURE 10-A

FIGURE 10.

IV

SECOND PROBLEM IN PERSPECTIVE

OUR second problem in perspective is much like the first, and except for a few points, may be drawn very easily by the student if he has mastered the other. The problem is to make a perspective drawing of the monument shown in Figure 6, but under different conditions than those of Figure 8. As stated below and shown in Figures 10 and 10*A*:

The picture plane shall be drawn through the corner of the base-step of monument, and the point of station shall be thirty feet from this point; the axis shall make an angle of 115° with the long side of base-step; the point of station and the horizon to be twenty feet above the ground. Under these conditions will be produced a so-called bird's-eye perspective. (See Figures 10 and 10*A*.)

The making of this perspective involves no new principles in so far as the monument is concerned; this should be done in exactly the same way as formerly. The large platform and steps upon which the monument stands will involve something new, however, for these are partly in front of the picture plane. The finding of the perspective of points lying in front of the picture plane has been lightly touched upon in the finding of a'' and q'' in Figure 8. We must now take this matter up very carefully, for this problem often occurs in perspective and is usually very puzzling to the beginner.

We must first make a diagram, Figure 10*A*, similar to Figure 7, to find *V*, *V′*, *M*, *M′*, and *C* according to the new conditions. Having found these, we lay off *H–H* for the picture and on it locate *V*, *V′*, *M*, *M′* and *C* as before. We then draw through *C*, the line of measures, *C*–18–*G′*, and on it we place, according to hypothesis, *G′*, twenty feet below *C*. We next locate the point 18, eighteen inches above *C′*; this gives us the bottom point of base-step of monument.

The perspective plan of the monument proper may be drawn by precisely the same methods as we used to draw the perspective plan in Figure 8. After locating 18′ conveniently on the line of measures and drawing through it the line *P′P′* and lines to *V* and *V′*, 18′–*X* and 18′–*Z*, we can lay off the scaled distances on *P′P′*, foreshorten on 18′–*X* and 18′–*Z* to *M* and *M′*, and draw lines through the newly found points to *V* and *V′*. Our next concern is to find such points as *S*, *C′*, *b* and *B*. We must remember two things: first, that we lay off actual distances on *P′P′* and by foreshortening to *M* we get the perspectives of these distances on S–18′–X, by foreshortening to M' we get the perspectives of these distances on b–18′–Z; second, that the measuring points M and M' may be used to find the perspective distances only on lines like S–18′–X and b–18′–Z, lines drawn to V and V' through 18′, the projection or plan of C. We can readily see by a glance at the perspective plan of Figure 10 that if we locate S and X, Z and b, and through them draw lines to V and V', these lines will be the sides of the platform, and their intersections the corners. X and Z may be found in the usual way by locating X' and Z' on $P'P'$, drawing lines through them to M and M', intersecting 18′–20′–X at X and 18′–22′–Z at Z.

Point S lies in front of the picture plane to the left of 18′. We lay off S', five feet to the left of 18′ on $P'P'$. Since S lies on S–18′–X, we draw a line through S' to M, intersecting S–18′–X at S. In the same way we find a'', the perspective of a', three feet from 18′.

We now find b, which lies in front of the picture plane and to the right of 18′. We lay off b', nine and one-half feet to the right of 18′ on $P'P'$; and since b lies on b–18′–Z, we draw line through b' to M', intersecting b–18′–Z at b. Likewise we find p, q and r, the perspectives of p', q' and r', nine, eight and seven feet respectively from 18′. We place g' on $P'P'$, two feet from X', and foreshorten to M on 18′–X to find g''. We now draw lines through S, a'', g'' and

X to V, and through Z, r, q, p and b to V', to complete the perspective plan.

On the perspective itself we already have 18 and G'. Through 18 and G' we draw lines to V'.

Draw the verticals $1'$–G'' and $2'$–a''' over S and a'' of plan and intersecting the lines through 18 and G in $1'$, $2'$, G'' and a'''.

On the line through 18 to V' locate $3'$ and $4'$ over g'' and X of plan. Draw verticals through $3'$ and $4'$, locating $12'$ on line through G'.

Six inches and twelve inches below 18 on line of measures, place $7'$ and $8'$, and through $7'$ and $8'$ draw lines to V', locating 9 and 10 on vertical $3'$–$12'$ and $7''$ on $1'$–G''.

Through $1'$ and $4'$ draw lines to V.

Draw verticals 16–17, 1–G, 4–B' over A, C', B of plan, and through 16, 1 and G draw lines to V'.

Draw verticals 2–aa and 3–g''' over a and g of plan, and through 2 and 3 draw lines to V.

Draw vertical 6–15 over $6'$ of plan, and through 6 draw line to V', intersecting 2–5 at 5; 5 should be over $5'$ of plan.

Draw verticals 13–14 and 11–12 over $13'$ and $11'$ of plan.

Through 9, 10 and $12'$ draw lines to V, locating 13, 14, 11 and 12. The line through 12 should intersect 3–g''' at g'''.

Through 15, 13, 14, 11 and 12 draw lines to V', the edges of the steps.

Through $7''$ draw line to V, locating 17; draw curve of terrace free-hand. This completes the platform and steps.

The problem should be gone over thoroughly and drawn without the instructions as many times as necessary to fix thoroughly in the mind the method of finding the perspective of points when they lie in front of the picture plane. No other one thing in perspective seems to bother and puzzle the beginner as much as this.

We locate a point in perspective by considering the point as the intersection of two lines which are parallel to and at a known distance from two already known lines. For example, *C′* of the plan in Figure 10 is the intersection of *C′A* and *C′B*, parallel to 18′–*Z* and 18′–*X* and five feet and nine and one half feet from these lines respectively. 18′–*Z* and 18′–*X* are perpendicular to each other in reality, so *S*–18′, five feet, is the perpendicular distance of *S* from 18′–*Z*. A line drawn through *S* to *V* is parallel to 18′–*Z*, which is also drawn to *V*. Likewise, *b*–18′ is perpendicular to 18′–*X* and nine and one half feet from it. *C′*–*b*–*B* drawn to *V′* is parallel to 18′–*X*, also drawn to *V′*. The intersection of these two lines gives us *C′*. So we may find any point in perspective. The two lines 18′–*Z* and 18′–*X* may be called the fundamental lines of the perspective. They are the starting lines, the lines upon which we build our perspective. To find any point in perspective, let us look at our actual plan, and after determining the two fundamental lines, usually the lines of the two principal systems of lines used—that is, those going to the two principal vanishing points—passing through the point which is the plan of the line of measures, we draw lines through the point in question, parallel to the fundamental lines, and find the distance of these lines from the fundamental lines; we can then, as we did a little while ago, draw the perspective of these lines, and their intersection will be the perspective of the point desired. We can thus find the perspective of any point. By finding the perspective of any two points on it, we can find the perspective of any straight line. By finding the perspective of enough points on it to define it, the perspective of any line, whatever its curve, may be found. So, in fact, if we can find the perspective of any point, we can find the perspective of any line, and therefore any object, by the finding of its defining lines. When we determine a line by drawing through a known point on it to the vanishing point, we determine the line by two points on it—one the point through which we draw, and the other the vanishing point. The whole science of perspective may be boiled down to the finding of the perspective of any point in space. The student should understand this very clearly, and by this time should be able to find the perspective of any assumed point under any conditions.

The student should examine his drawings, Figures 8 and 10, very carefully and note the different ways in which the same point may be found. Let us take the point 5, Figure 10. We found it by drawing from 6 to V' and from $2'$ to V, intersecting at 5. We might have found it by going from r''' to V' and from $2'$ to V, intersecting at 5; or by drawing from 2 to V and placing 5 on this line over $5'$ of plan, likewise by drawing from 6 to V' and placing 5 over $5'$. There are usually many ways of locating the same point, although all these ways amount to the same thing and give the same result. The student, with practice, will use the most convenient way almost intuitively. It is excellent practice to analyze both Figures 8 and 10 to discover the various ways in which the principal points may be projected and located.

V

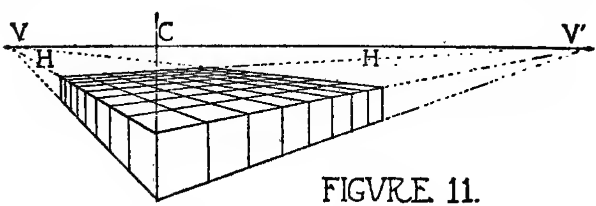
VANISHING POINTS AND THE POINT OF STATION

THE fundamental point of a perspective is obviously the point of station. This point, as its name implies, is the one where is the observer's eye. Upon its location depends, more or less, the position of every working point in the drawing. The point of station has, first, a plan position and, second, a position of elevation or height. Its plan position determines the distance apart of the vanishing points and measuring points. Its elevation determines the height of the horizon. As has been said before, the proper choice of the point of station determines the artistic value of the picture, and the ability to make this choice properly depends almost entirely on the judgment born of experience and observation of nature. As a hint for architectural perspective, it is usually best to avoid locating the point of station so that the obliquity of the two sides of a building is the same; it is usually better to have one side—if there is any choice, the less important one—foreshortened very perceptibly more than the other, giving pleasing contrast to the angles of the drawing.

The height of the point of station, as has been said before, determines the height of the horizon. The horizon is always at the level of the eye of the observer. If he moves up, the horizon moves up with him; if he moves down, it moves down with him. This is obvious and a matter of daily experience to all of us. The point of station should not be taken too near the object. If it is, the vanishing points and measuring points are too close together and the whole perspective looks forced and strained—we get the same artificial effect as we do in photographs taken with an extremely wide-angle lens. Neither should the point of station be taken too far away, unless a telescopic or very distant effect is desired. Its distance should depend on the size of the object. A good way is to place the point of station at such distances as to make lines drawn from the extremities of the object to it about 50° or 60° apart. From the artistic standpoint, the point of station is not well located in Figure 10. It was placed as it was simply to bring out some new principles in perspective, and to do so without sacrificing compactness of drawing.

We must note that a line may decrease in perspective from two causes: one, the angle it makes with the picture plane, and again on account of its distance from the point of station. The greater the angle with the picture plane, the shorter the line becomes in perspective; the greater its distance from the point of station, the less its perspective length. Both of these facts depend on the principle previously given, that the size of an image depends on the angle subtended at the eye (Figure 3). Both these facts are well illustrated in Figure 2. The principal windows in the photograph of Figure 2 on both ends and front are the same width, yet the end ones are much narrower in the picture than the front ones because the end makes a much larger angle with the picture plane than the front. The windows on the front at the extreme right are narrower than those at the left because of their greater distance from the point of station.

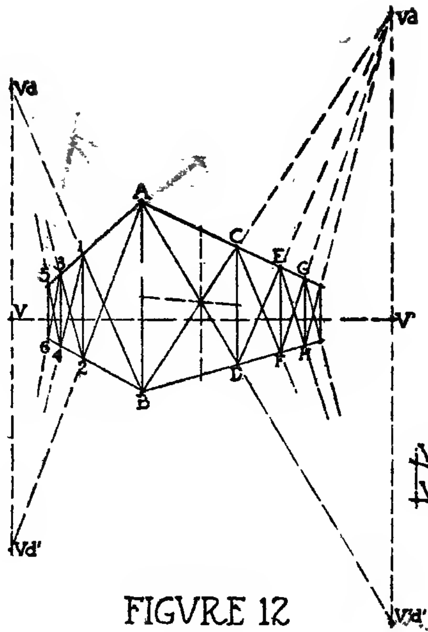
We should also notice an apparent crowding of lines as we approach the vanishing points or the horizon. This can be seen to some extent in Figure 2, but more easily in Figure 11, where



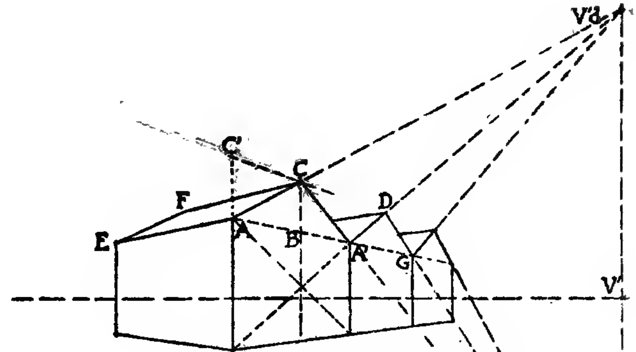
FIGVRE 11.

the first three squares on either side and the top occupy more space than the other five. There is an apparent crowding of lines on top towards the horizon and at the sides towards the vanishing points.

In Chapter II we explained that two parallel lines have a common vanishing point on account of the apparent decrease of distance between them, this distance becoming infinitesimal, or a point. Let us consider two parallel planes. Let us imagine in these parallel planes two parallel lines, one in each plane, having, of course, a common vanishing point. Let us now imagine another pair of lines oblique to the first but parallel to themselves; these will also have a common vanishing point. So we can imagine many pairs of lines and get a series of vanishing points. These vanishing points will define a line, and this line is the vanishing line of the two parallel planes. So each system of parallel planes has a common vanishing line, and all parallel lines lying in these planes have their vanishing points in the vanishing line of the planes. This is an important principle in perspective. It is for this reason, as has already been explained in Chapter II, that all horizontal lines vanish in the horizon, the vanishing line of horizontal planes. A vertical plane, oblique to the picture plane, will have a vertical vanishing line. If in this vertical plane we draw a horizontal line, its vanishing point will be in the horizon. A vertical line drawn through this vanishing point will be the vanishing line of the plane, because vertical planes must meet in a straight vertical line; the vanishing point of any line in these planes must be in the vanishing line of these planes; and since but one vertical line can be drawn through any given point, the vertical line drawn through the vanishing point of the horizontal line, and which point is in the vanishing line of the vertical plane, as above stated, must be the vanishing line of the vertical plane. By this principle we may find the vanishing points of oblique lines lying in known vertical planes, such as gable lines, diagonals, inclined grade or sidewalk lines against buildings, and so on. In Figure 12, 1–2–*A*–*B*–*C*–*D* is the perspective of a cube. A vertical line, *V′d*–*V′*–*V′d′*, through *V′* is the vanishing line of the plane *ABCD*. If we draw the diagonal *BC* in this plane, its vanishing point must lie in the vanishing line of its plane, or *V′d*–*V′*–*V′d′*, so if we continue *BC* to its intersection, *V′d*, with this vanishing line, we get *V′d*, its vanishing point. By the use of this vanishing point we may draw the diagonal *DE*, which is parallel to *BC*, thus locating *E;* draw the vertical *EF*, then the diagonal *FG*, and so on. We draw any number of successive rectangles in perspective without the use of the perspective plan after we have drawn the first and its diagonal. Likewise, the point *Vd* may be found and the squares or rectangles on the other side drawn. It is also obvious how *V′d′* and *Vd′* may be found and used similarly. Figure 12*A* shows an application of this principle to architectural perspective in drawing a triple-gabled house. We may



FIGVRE 12



FIGVRE 12A

FIGVRE 12B

find the distance *AB* by perspective plan in the ordinary way, draw the vertical *BC*, finding *C* from *C′*, thus getting the height of the gable. We may now draw the gable line, *AC*. By drawing the vanishing line *V′d–V′–V′d′* and continuing *AC*, we find *V′d*, the vanishing point of *AC* and its parallels. We may similarly find *V′d′*, the vanishing point of *CA′* and its parallels. We may now draw *EF* and *A′D, DG* and so on. The same method is used in drawing such inclined planes as sidewalks, streets and roads when these are not horizontal (Figure 12*B*). One line can always be established, and its vanishing point, corresponding to *V′d* or *Vd*, found; the lines parallel to it may then be drawn at the proper distances.

There are many other applications of this principle, and the occasions for its use are almost innumerable.

The vanishing point of any line may be found by looking along that line; this is obvious, for if we look along a line its image is a point—its vanishing point. The line whose vanishing point we wish to find, however, may not pass through the point of station, so we cannot look along it. If, then, we look along a line passing through the point of station and parallel to the first line, we easily find the required vanishing point, for we find the vanishing point of its parallel, and parallel lines have common vanishing points. This is why, in Figures 7 and 10*A*, we draw lines through the point of station, parallel to the sides

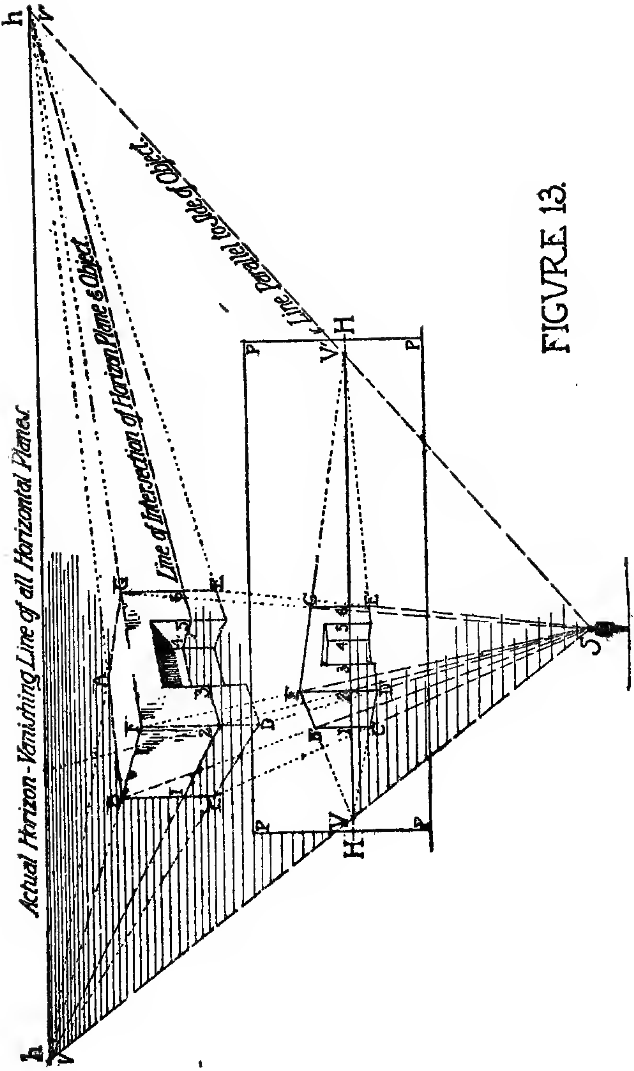

Actual Horizon-Vanishing Line of all Horizontal Planes
Line of Intersection of Horizon Plane & Object
Line Parallel to Side of Object

FIGURE 13.

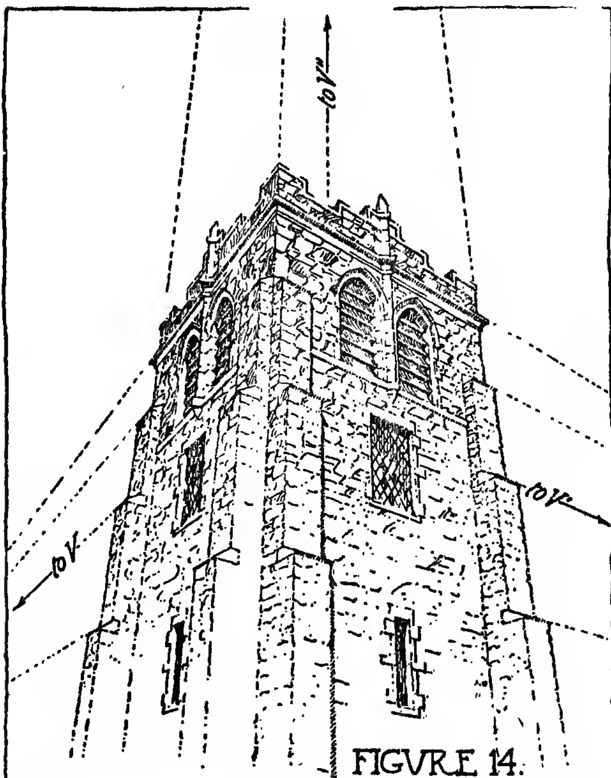
of the object, to get the vanishing points of lines in the sides of the object. This is further shown in Figure 13, where *A–B–C–D–E–F–G* is an object in space; *hh,* its actual horizon, and *vv′,* the actual vanishing points of its two systems of lines; *PPPP,* the picture plane; and *S,* the point of station. The lines *Sv* and *Sv′* are drawn through *S* parallel to the systems of lines of the object, hence going to the vanishing points, *v* and *v′*. These lines intersect the picture plane (represented by the line *PP′* in Figure 7) in *V* and *V′,* the vanishing points of the picture. These lines, obviously, lie in a horizontal plane through *S,* which intersects the picture plane in the horizontal line *HH,* the horizon of the picture. *V* and *V′*, points in this horizontal plane and in the picture plane, must lie in the intersection of the two planes, *HH.* This figure tells its own story quite plainly, and should be very carefully studied by the student; the more he studies it, the more procedures in his making of a perspective will be explained.

When you looked along a line to see its vanishing point, it perhaps did not occur to you that there was a vanishing point for that line behind you as well as in front. If you were looking north along a north and south line, you would see the vanishing point of the line in the north, but there would also be a corresponding vanishing point for the same line in the south, 180° away. This second vanishing point is almost never used, and is of practically no importance.

In panoramic or curvilinear perspective—a subject which we need not discuss here—an attempt has been made to utilize the two vanishing points of a line. Note, in a panoramic picture when flattened out, the convergence or curvature of horizontal lines to points at each end.

Special relations sometimes exist between vanishing points, and some of these we shall now discuss. When two points are the vanishing points of two systems of lines which are perpendicular to each other—that is, when the direction of one system is perpendicular to the direction of the other—then the vanishing points are called conjugate vanishing points. Most cases of architectural perspective employ conjugate vanishing points because the planes of buildings are usually perpendicular to each other, hence the line systems are perpendicular to each other. Of course this is not always the case, and there is really little use in differentiating conjugate vanishing points from others. There are also reciprocal vanishing points, which are the corners of an equilateral triangle of which the point of station is the apex; they are 60° apart and are sometimes used for the drawing of hexagons in perspective under special conditions. The reader at this stage of his study need not trouble himself about them.

The most interesting of special cases is the one of tri-conjugate vanishing points. If we place a rectangular block on one of its corners, so that none of its line systems is parallel to the picture plane, then there will be three systems of lines perpendicular to each other and having three vanishing points and three horizons. The same will occur if the object or block is placed in the ordinary way and the picture plane is in-



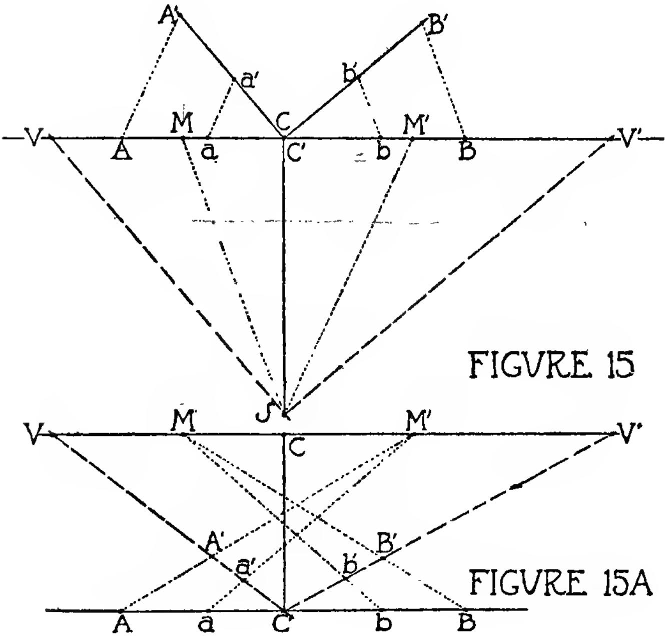
FIGVRE 14.

clined or out of plumb or vertical. The verticals on the object converge in the perspective; this happens in a distorted photograph when the camera has been tilted up or down and the ground glass has not been made vertical. We have all seen such photographs. The three vanishing points in such cases are conjugate in pairs and the whole group is called tri-conjugate. Tri-conjugate vanishing points are not of any great practical use, and need not be taken up by the student now. (See Figure 14.)

VI

MEASURING POINTS AND SCALES

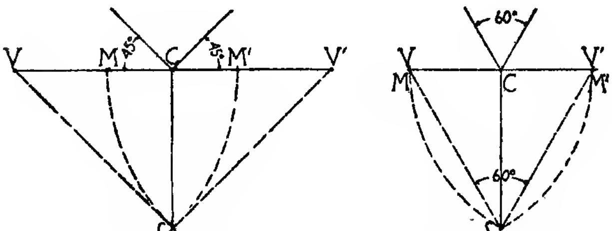
AS has already been shown, there are lines in a perspective drawing which are their own perspectives and on which distances may be scaled. Such lines must lie in the picture plane and may be called the scales of the perspective. It may be mentioned here that lines which lie in the picture plane and are their own perspectives are called front lines, and all perspective scales are front lines. Such lines are the line of measures and $P'P'$ in Figures 8 and 10. To throw scaled distances in perspective, we must draw to the vanishing points, or to certain points called measuring points (which are special vanishing points), or to both. We stated in Chapter III that the measuring points were the vanishing points of parallel lines which made equal angles with a front line and another line passing through the center C, or, as shown in Figure 15, lines that make equal angles with CB' and CB or with CA' and CA. In Figure 15, which is a diagram similar to Figure 7, let us draw the parallels BB' and bb' so that the angles at B, B' and *b*, *b′* are equal; this is easily done by making *CB* equal to *CB′* and *Cb* equal to *Cb′*. The triangles



FIGVRE 15

FIGVRE 15A

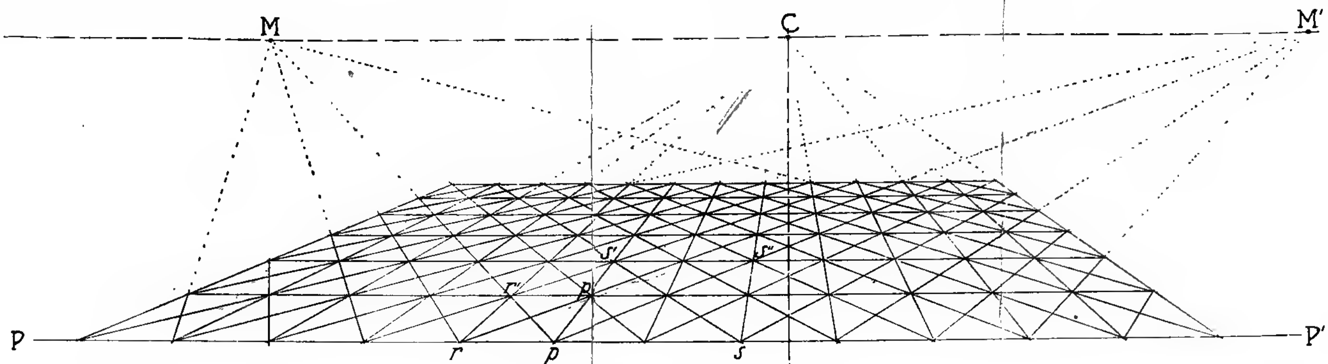
BCB′ and *bCb′* are then isosceles, and their base angles are equal. We now draw *SM* through *S*, the point of station, parallel to *BB′* and *bb′*; the point *M* will then be the vanishing point of *BB′*, *bb′* and their parallels. *SV′* was drawn parallel to *CB′* and *SM′* is parallel to *BB′*, therefore the two triangles *CBB′* and *V′SM* are similar geometrically, their angles being homologously equal; that is, the angles MSV' and SMV' are equal to the angles $BB'C$ and $B'BC$, but the angles $BB'C$ and $B'BC$ are equal to each other (drawn so), so the angles MSV' and SMV' must be equal to each other, and the triangle $SV'M$ is isosceles. The sides of this triangle, MV' and SV', must therefore be equal, or M is the same distance from V' as S is from V'. Likewise, it may be shown that VM' is equal to VS, or that M' is the same distance from V as S is from V. Figure 15 is the plan or orthographic projection of the perspective plan, Figure 15*A*, or rather the perspective plan is the perspective of Figure 15. Figure 15*A*, which is lettered similarly to Figure 15, illustrates this. In Figure 15*A*, $AC'B$ is a front line, and the perspective plan of the picture plane; the line VCV' is also in the picture plane and is the horizon line of the perspective. In the orthographic projection or plan of Figure 15, these two lines coincide. The triangle $BC'B$ is the perspective of the isosceles triangle of Figure 15, and M is the vanishing point of BB' and its parallels, which cut off distances perspectively equal on $C'V'$ and $C'B$. This has been explained before. The angles B, B', b and b' are perspectively equal. The triangles formed by $C'V'$ and $C'B$ and lines drawn to M (Figure 15*A*) are always perspectively isosceles, and so M is called a measuring point because a line drawn to it from a point on $AC'B$ will cut off or measure the perspective of the distance of the point from C' on $C'V'$. All this is similarly true of M'. It was proved in Figure 15 that MV' equals SV' and that $M'V$ equals SV. This is always true, under all conditions; the measuring point for a line is the same distance from the vanishing point of that line as the vanishing point is from the point of station.



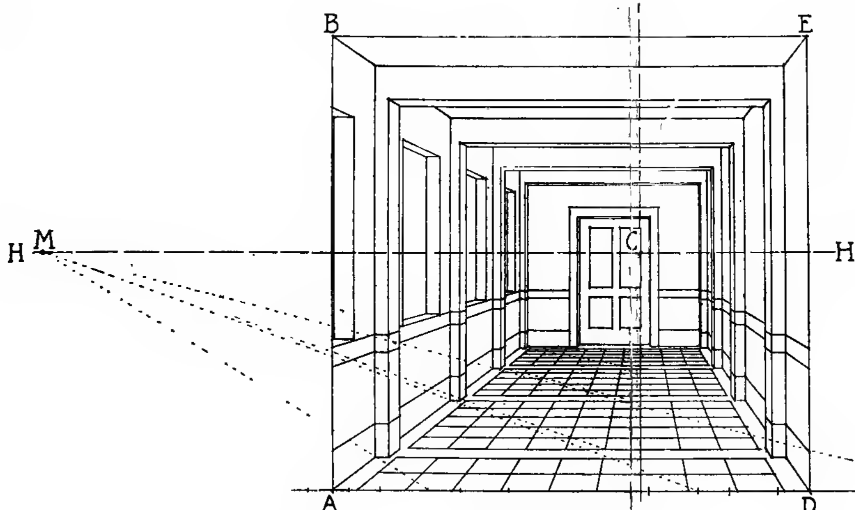
FIGVRE 16A. FIGVRE 16 B

When the center, C, is midway between two conjugate vanishing points (Figure 16A), the measuring points are symmetrically placed; that is, MV' equals $M'V$, and the distance of either measuring point from the corresponding vanishing point is equal to the diagonal of a square whose side equals one half the distance between vanishing points.

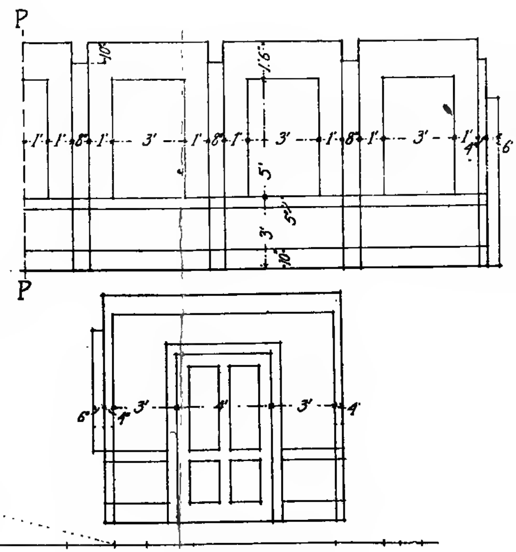
When the center, C, is midway between two reciprocal vanishing points (Figure 16B), then each vanishing point is the measuring point for the other vanishing point. This is obvious because VSV' is an equilateral triangle and VV' equals $V'S$ or VS. These special cases of measuring points are of no great practical use, but are of interest as exemplifying some curious geometric relations in perspective.



FIGVRE 18A



FIGVRE 18B

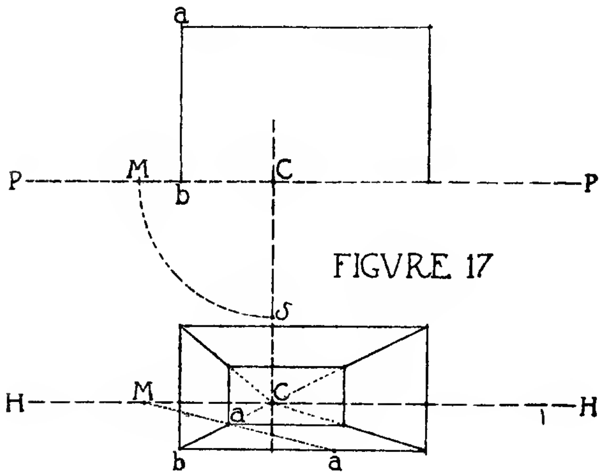


FIGVRE 18C

VII

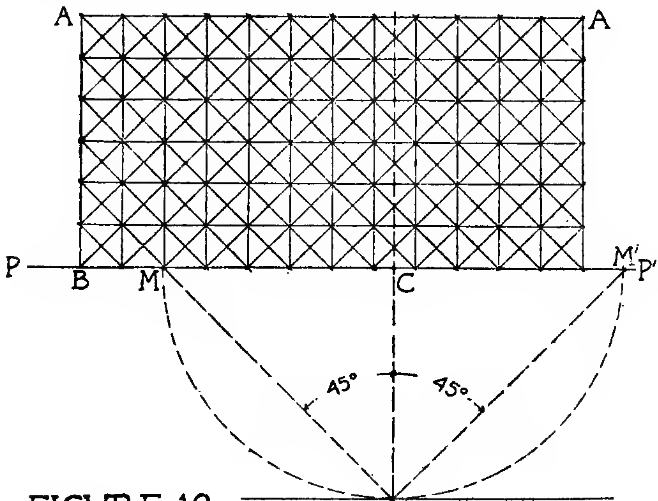
PARALLEL OR ONE-POINT PERSPECTIVE

THERE are cases in perspective where the picture plane is assumed parallel to one side of the object. The vanishing point for lines in this system is at infinity; in other words, all the lines in the system retain their original direction, and horizontal lines remain horizontal, just as vertical lines remain vertical, and for the same reason. There is one finite vanishing point for the other system of lines. This vanishing point is, of course, on the horizon opposite the point of station; it coincides with the center, *C*. This is evident from the diagram in Figure 17. The measuring point—we usually use but one—is located in the same way as previously explained for two-point perspective; *MC* is equal to *CS*. The measuring point is always as far away from the vanishing point as the vanishing point is from the point of station. The resultant perspective, made under the conditions just described, is called a parallel or one-point perspective. One-point perspective is useful in many cases, but especially for the perspectives of interiors or street vistas. It must



FIGVRE 17

be remembered that it should not be used where the width of the picture is great; it then becomes forced or artificial. It should be confined to pictures where the angle of view, the angle between lines drawn from the extremities of the object to the point of station, is not over 50° or 60°—the less the better. Figures 18*A* and 18*B* show parallel perspectives. Figure 18*A*, a parallel perspective of a series of horizontal squares with their diagonals, very lucidly illustrates the principles of one-point perspective. It should be noticed in this drawing that the measuring points, *M* and *M′* (placed so that *MC* or *M′C* equals *CS*), are the vanishing points of the diagonals of the squares or lines at 45° with the picture plane. This will be readily seen if we look at one-point perspective simply as a special case of two-point perspective as shown in Figure 19.



FIGVRE 19.

In Figure 19 we draw the plan of the squares of Figure 18*A*, and locate the point of station, *S*. *PP′*, the plan or trace of the picture plane, coincides with the front of the first row of squares, one system of lines being by hypothesis parallel to the picture plane, and the other perpendicular. If we, then, proceed to find the vanishing points, we draw through S a line parallel to the AA system of lines. We get no finite vanishing points, for the line will intersect PP' at infinity, being parallel to it. If, again, we draw a line through S parallel to the AB system, the line will be perpendicular to PP' and will intersect it at C, the center. The points M and M' being CS distant from C, the lines MS and MS' are at 45° with CS and PP'; they are parallel to the diagonals of the squares, which, of course, are at 45° with PP', therefore M and M' are the vanishing points of these diagonals.

Now let us turn again to Figure 18*A*. Since M and M' are the vanishing points of the diagonals of the squares, we can easily see that if through any point, p, on PP', we draw a line to C, and to the right of p lay off a point, s, and if from s we draw a line to M, the distance $s'p$ will be perspectively equal to ps, for ss' is the diagonal of the square whose equal sides are $s'p$ and ps. Likewise, if we draw r to the left of p and draw a line to M', $p'p$ will be perspectively equal to pr, for rp' is the diagonal of the square whose equal sides are pr and $p'p$. Also, if we draw lines to C through r and s and then draw horizontal lines through p' and s', rr' will equal $p'p$ and $s''s$ will equal $s'p$ perspectively, for we shall have merely completed the perspectives of the squares.

This shows briefly the whole theory of one-point or parallel perspective, simply a special case of two-point perspective with conjugate vanishing points, but having one vanishing point at infinity on account of one system of lines being parallel to the picture plane and the other vanishing point coinciding with C on account of its system of lines being perpendicular to the picture plane. Of course, lines whose vanishing points are at infinity are parallel.

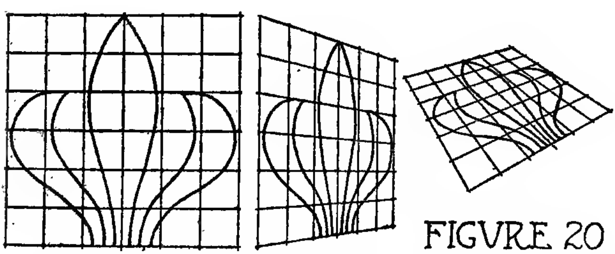
Figure 18*B* shows a one-point perspective of a beamed corridor, the elevations of which are shown in Figure 18*C*. This figure should be redrawn by the student. In Figure 18*B*, the horizon, *HH*, is drawn and *C* and *M* located on it, then the rectangle *ABDE* is drawn to scale, a section of the corridor, and lines from *A*, *B*, *D* and *E* are drawn to *C*. Actual distances on the side wall are laid out on the ground line, *AD*, to scale and foreshortened on the perspective line through *A* by drawing lines to *M*. These distances may be transferred to the opposite side or any opposite vertical by horizontal lines. Vertical distances or heights are laid out on *BA* or *ED* and foreshortened to *C*. On the ground line, *AD*, we actually project a one-point perspective plan, the floor plan, to locate all our verticals. This plan may be drawn at another level below or above our picture, just as we did in our other perspective problems, but in parallel perspectives of interiors it is usually more convenient to use the perspective of the floor itself as the perspective plan.

It must be understood that parallel or one-point perspective is not confined to interiors. Any object may be drawn so; usually, however, interiors and street vistas, especially narrow ones, are the only pictures that appear at all well. Where the horizontal lines become inordinately long, the picture looks artificial. For interiors within reasonable limits, parallel perspective is very convenient and considerably simpler than two-point.

VIII

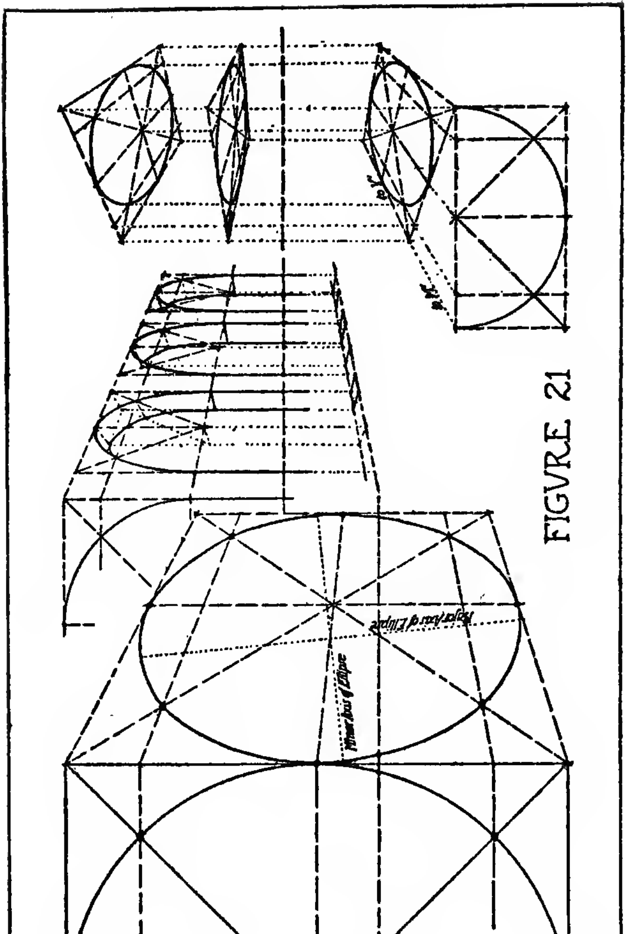
SPECIAL MANIPULATIONS AND SHORT-CUTS

THE perspective center of any rectangle may be found by drawing the diagonals of its perspective. This is quite obvious. Two straight lines can intersect in but one point; the diagonals of a rectangle intersect in its center, therefore the intersection of the perspectives of the diagonals is the perspective of the center. (See Figure 12.)



FIGVRE 20

Very often the perspectives of very irregular outlines, irregular curves, patterns of various

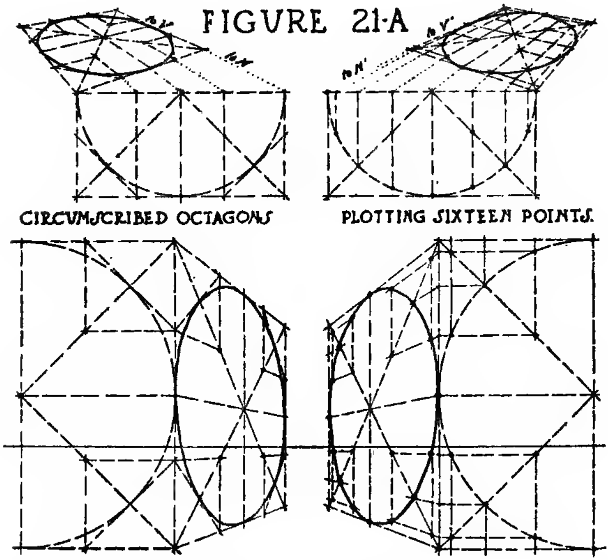


FIGVRE 21

sorts, landscape plans, and so on, have to be drawn. The simplest way to do this is to plot geometrically the orthographic plan or projection of the outline to be drawn, then throw the plotting diagram into perspective. We can then locate the intersections of the outline with the plotting reference lines and obtain enough points to determine the perspective. This process is called craticulation and is quite apparent from Figure 20.

The drawing of circles and other curves in perspective is based on the process of craticulation explained above, but there are special cases where the defining points on a circle can be easily located. Usually we draw the circumscribed square and its diagonals and find the points of intersection of the diagonals and circles; this will then give us the eight points on the circle—four tangent points and four intersections on the diagonals. A close inspection of Figure 21 will make this method of drawing circles and their arcs in perspective quite plain. It is very difficult for the beginner to draw the perspectives of circles neatly, especially in horizontal planes. The perspective of a circle is a line of great beauty, being a true ellipse, but we must remember that the perspective of the diameter of the circle is not the axis of the ellipse. This is also shown in Figure 21. We should be very careful of the points of tangency or change of direction in our perspective circles. The ends of the ellipse are usually a great deal sharper than we are inclined to draw them. A close study of nature, painstaking analysis of correct photographs, and vigilant practice are necessary to acquire skill.

FIGURE 21-A



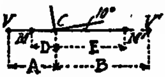
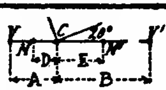
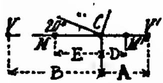
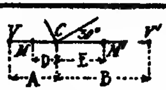
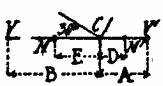
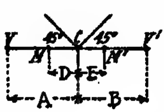
First drawing the circumscribed octagon often helps to determine the proper curve of the circle in perspective. Just as we found the eight points on the circle in Figure 21, so may we, by the same method, find the eight corners of the circumscribed octagon. The left side of Figure 21*A* shows this clearly. In addition to the circumscribed square and its diagonals, we draw the octagon inscribed in the square and circumscribed about the circle. If we construct this in elevation on a line in or parallel to the picture plane as shown, the necessary points are easily found in perspective. The eight sides of the octagon, all tangent to the circle, are a distinct aid in guiding the direction of curvature in the perspective circle.

By the same general method of using points and their ordinates in the picture plane or one parallel to it, sixteen or more points on the circle may be easily found, as shown at the right of Figure 21*A*. This is useful when large circles are drawn in perspective, and may, of course, be used in combination with the circumscribed octagon, if so desired. These methods can be much more plainly demonstrated graphically than described in words, and a study of Figure 21*A* should make everything quite plain.

When the principal line systems of a perspective are perpendicular to each other, as is usually the case, and the vanishing points are conjugate, it is very useful to locate the vanishing points of lines which bisect the right angles of the plan—lines which are at 45° with the principal systems. Lines drawn to these points are very useful in drawing the perspective plans of hip roofs, the projections of cornices, and for finding shadow projections. These special vanishing points are called miter points, and lines drawn to them are called miter lines. When used in the casting of shades and shadows they

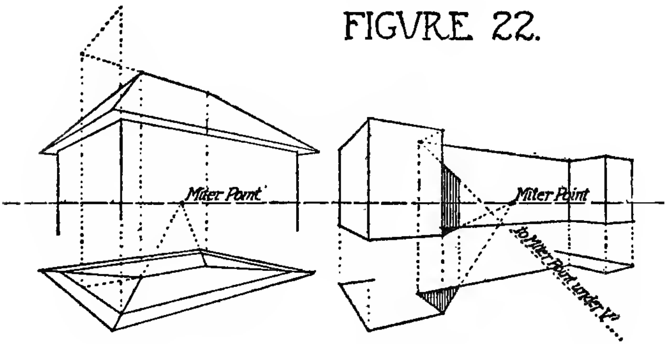
TABLE OF CONJUGATE VANISHING POINTS WITH CORRESPONDING MEASURING POINTS FOR DIFFERENT ANGLES OF VIEW.

ALL DISTANCES ARE INDICATED IN INCHES.

ANGLE	V–V'	A	B	D	E
	20	5/8	19 3/8	5/16	2 7/8
	30	15/16	29 1/16	15/32	4 5/16
	40	1 1/4	38 3/4	5/8	5 3/4
	50	1 9/16	48 7/16	25/32	7 3/16
	20	2 3/8	17 5/8	1 1/8	4 1/2
	30	3 9/16	26 7/16	1 11/16	6 3/4
	40	4 3/4	35 1/4	2 1/4	9
	50	5 15/16	44 1/16	2 13/16	11 1/4
	20	5	15	2 5/16	5
	30	7 1/2	22 1/2	3 15/32	7 1/2
	40	10	30	4 5/8	10
	50	12 1/2	37 1/2	5 25/32	12 1/2
	20	10	10	4 1/8	4 1/8
	30	15	15	6 3/16	6 3/16
	40	20	20	8 1/4	8 1/4
	50	25	25	10 5/16	10 5/16

BY PROPORTION A, B, D AND E MAY BE FOUND FOR ANY ASSUMED LENGTH OF V——V' are sometimes called shadow points and shadow lines. These various points are illustrated in Figure 22.

FIGVRE 22.



It often happens in large perspective drawings that a vanishing point is so far away as to be off the drawing-board. In that case it is usually best to use a "curve" and specially arranged T-square. If we draw any chord of an arc of a circle and draw the perpendicular bisector of that chord, this bisector will pass through the center of the circle. (See Figure 23*A*.) Using this principle of geometry, we may easily draw lines to a vanishing point any distance away. In Figure 23*B*, *AA* is an arc of thin wood or other material similar to the railroad curves used by engineers and sold at drawing-material supply-stores. *B* is a T-square which has the one edge of its blade in the center of the head. When the head of this T-square is placed

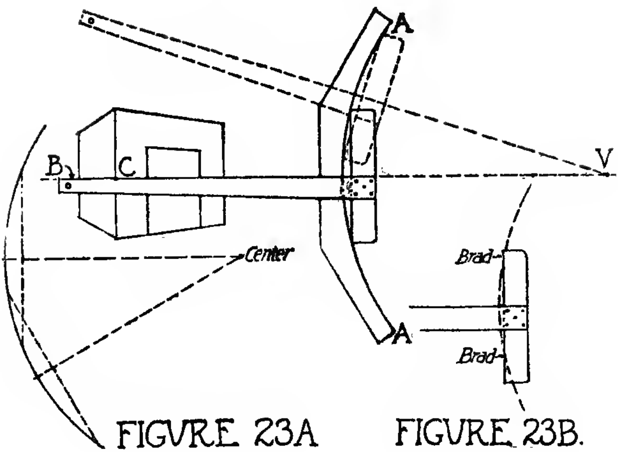


FIGURE 23A FIGURE 23B.

against the concave side of the curve, its edge may be taken as a chord of the arc, *AA*, and the central edge of the blade as the perpendicular bisector of the chord, hence the central edge of the blade will always point to the center of the arc when the head is placed against it. If we assume *V* as the vanishing point of a perspective and place *AA* so that *V* is its center, the T-square will locate lines to *V* if its head is made to follow the curve. To place the curve properly, we must first draw on it a radial line. This is easily done with the T-square, for when its head is placed against the curve, its central edge of blade marks a radial line. The horizon line of the perspective is a radial line of any arc drawn with *V* as a center. If, then, we place the arc, *AA*, so that the radial line on it coincides with the horizon line and so that it is its radius distant from *V*, the T-square will draw lines converging to *V*. In placing the arc, of course we do not need *V;* we simply need to know the distance of *V* from *C;* if from this distance we subtract the radius of the arc, the remainder will give us the distance of the arc from *C*. In practice we need not buy the manufactured railroad curves spoken of above; they may be easily cut from thin wooden picture-backing. Any T-square, also, may be used by driving brads into the head, equidistant from one edge of the blade and equidistant from the working edge of the head, and running the brads on the curve. (See lower right-hand corner of Figure 23*B*.)

When we have but one or two lines to get and the vanishing point is off the board, it may be done very easily by what may be called proportional points. In Figure 24 it is desired to draw a line through *A* to *V*, which is off the board. We place *v* on *HH* so that *Cv* equals one fourth (any convenient ratio may be used) of *CV;* we place *a* on the line of measures so that *Ca* equals one fourth of *CA*. We draw a line through *a* to *v*, then through *A* we draw a line parallel to *av;* this line goes to *V*, for the lines *av* and *AV*, being parallel, will cut off proportional distances on *CA* and *CV*, or *CV* is to *Cv* as *CA* is to *Ca*, which is the correct proportion by hypothesis.

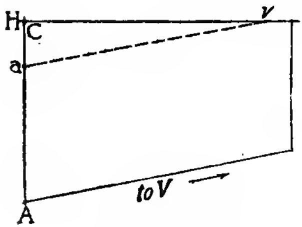


FIGURE 24.

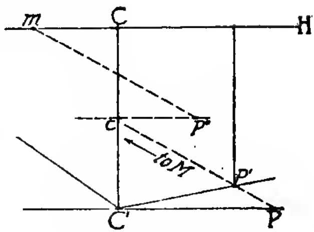


FIGURE 25.

This principle may likewise be used for drawing to measuring points which are off the board, as shown in Figure 25. Here we wish to draw a line from *P* to *M* to find its perspective, *p′*. We take *Cc* equal to one half of *CC′*, *Cm* equal to one half of *CM*, and *Cp″* equal to one half of *C′P*. We draw a line from *p″* to *m* and next draw a line from *P* parallel to *p″m*, giving us *p′*. This is proved geometrically in the same way as Figure 24, by proportion.

If *P* is off the board and we wish to find its

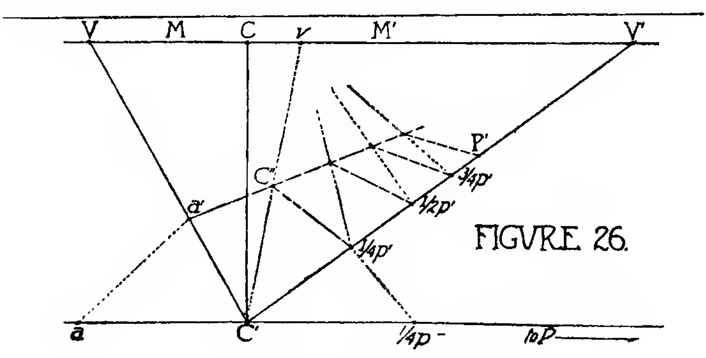


FIGURE 26.

perspective, P', we can do so as shown in Figure 26. We lay off $C'\frac{1}{4}p$ equal to $\frac{1}{4}C'P$, and find the perspective of $\frac{1}{4}p$ at $\frac{1}{4}p'$. We make aC' equal to $C'\frac{1}{4}p$, and find the perspective of a at a'. By drawing from a' and $\frac{1}{4}p'$ to V' and V, we draw the perspective square, $C'\frac{1}{4}p'C''a'$. We now find v the vanishing point of its diagonal. By drawing to v from $\frac{1}{4}p'$, we can complete the adjacent succeeding square, finding $\frac{1}{2}p'$, and so on until we locate p'. This is a very convenient and interesting use of the vanishing point of a 45° line, or the miter point spoken of earlier in the chapter.

In parallel perspective it sometimes happens that both P, the point of which we wish to find the perspective, and m, the measuring point, are off the board or inconveniently far away; then

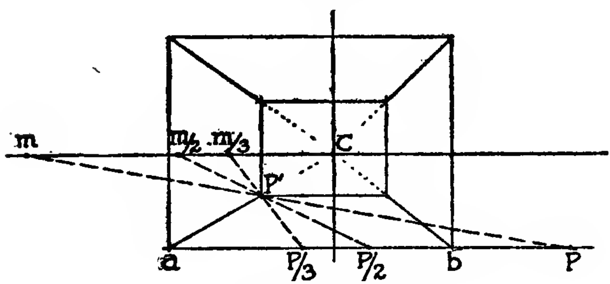


FIGURE 26A.

it is very convenient to use what are called fractional measuring points.

In Figure 26A, P is the point of which we wish to find the perspective, P', and m is the measuring point. If, instead of laying off the full distance, aP, on the horizontal line of measures, we lay off $P/2$, $1/2aP$ from a, or $P/3$, $1/3aP$ from a, and project, in case of $P/2$, to $m/2$, $1/2mC$ from C, or in case of $P/3$, to $m/3$, which is $1/3mC$ from C, we must get the same intersection, P', on aC. This is true, of course, for any ratio and may be readily proven by making an orthographic diagram or plan for Figure 26*A*, similar to Figure 19. This principle is very useful on account of its convenience, and may also be utilized in drawing perspectives without recourse to vanishing points, as explained in the next chapter. The same method may also be used in two-point perspective, remembering that the distance of the measuring point is always laid off from the corresponding vanishing point. The problems shown in Figures 25 and 26 may be easily solved by the use of these fractional measuring points.

It is sometimes desirable to limit the distance between vanishing points so as to get them on a certain drawing-board. If they are conjugate, this can be done by placing them the desired distance apart on the horizon line and constructing the point diagram as shown in Figure 27. We draw a semicircle on VV' as diameter; we locate C so as to give the proper angle of view; by drawing CS perpendicular to VV' we get S, the point of station on the semicircle. M and M' are found in the usual way. This construction depends on the geometrical proposition that every angle inscribed in a semicircle is a right angle; hence *VSV′* is a right angle and *V* and *V′* are conjugate vanishing points for the point of station, *S*, as long as it is on the semicircle. This method, though sometimes convenient and often used, is not the logical way to begin a perspective.

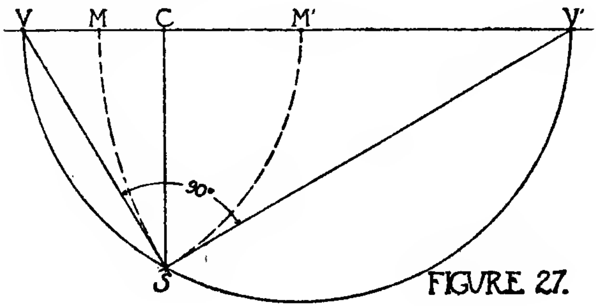


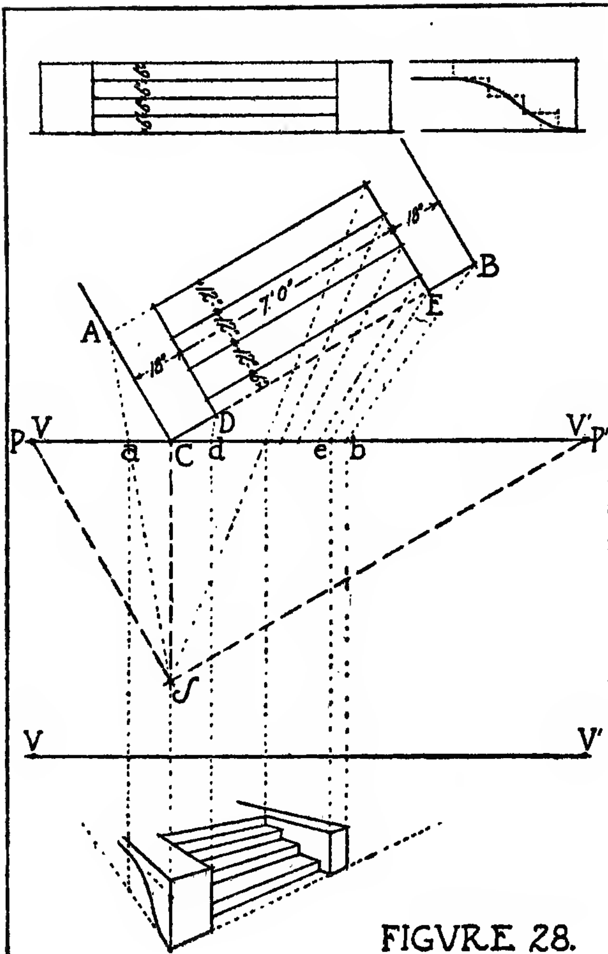
FIGURE 27.

Among draftsmen much is usually said of distortion, and a world of sins in perspective this word is made to cover. Within the legitimate meaning of the word there is no such thing as distortion in a correctly drawn perspeetive. If a perspective is accurately made, it is an accurate projection or image of an object as seen from one point—the point of station. If the eye, in viewing the picture, is placed at this point, the drawing appears correct and no distortion is apparent. Here lies the whole trouble: usually the picture is viewed from a point very much at variance with its point of station, and the drawing appears distorted, especially if it has been made under extreme conditions of vanishing points or point of station—when these have been inordinately close together, for instance. Here is another hint for the location of the point of station. It should be so located that the point from which the picture is to be viewed will not be very much at variance with it. This is, of course, very important in the hanging or placing of pictures. Place them so that they may be viewed as nearly as possible from their point of station. In looking at or sketching from nature, we are accustomed to view an object from several—sometimes many—view-points. We are constantly shifting our eyes, our point of station; but when we make a perspective, we record the image as seen from but one of these points, and when looking at this image, if the eye is not in the same relative position as the point of station for the image, we necessarily see things awry. So-called distortion is one of the shortcomings rather than the sins of perspective.

IX

VARIATIONS

THE preceding chapters were all based on the use of measuring points for the finding of perspective distances and lengths. To locate our verticals we used our plan in perspective, hence we may call it the method of "perspective plan." (It is often called the "Ware method" in this country, being named for Prof. Wm. R. Ware, who published his comprehensive treatise, "Modern Perspective," in the early eighties.) There is another method, precisely the same in theory but different in manipulation, using no measuring points but finding perspective lengths and distances by direct projection on the horizontal trace or plan of the picture plane. In this method we do not use a perspective plan, but the orthographic plan as shown in Figure 28, which will soon be explained. This method may be called the method of "direct projection," this being a slight misnomer, as direct projection is used only in the horizontal plane. The advantages and disadvantages of both methods will be taken up in a later paragraph.



FIGVRE 28.

In Figure 28, *ACB* is an actual plan or orthographic projection of a small flight of steps with bulkheads, *PP′* is the trace or plan of the picture plane, and *S* is the point of station. The vanishing points, *V* and *V′*, are found exactly as we found them in Figure 7, by drawing lines from *S* parallel to *AC* and *CB*, intersecting *PP′* in *V* and *V′*. We now draw lines from *A*, *B* . . . to *S* and get their intersections with *PP′* at *a*, *b* . . . These points, *a*, *b* . . . , mark the location on the picture plane of the perspectives of the verticals at *A*, *B* . . .

After projecting all these points to *PP′* such as *a*, *b*, *c*, *d*, *e* . . . , we may draw the verticals of our picture directly under them—or we may locate these verticals on our picture by transferring the distances *aC*, *Cd*, *Ce* . . . by a strip of paper or scale to our picture, just as we located the vertical lines over the corresponding points of the perspective plan in Figure 8 or Figure 10. After this the making of the picture is exactly similar to the method employed in Figures 8 and 10.

Figure 29 is a plan-diagram for the perspective of Figure 8, with points projected to trace of picture plane after the method of direct projection. If this diagram is redrawn to the scale of the perspective of Figure 8, the points *a* . . . *C* . . . *b* marked off on a strip of paper and laid over this perspective will be found to coincide with the corresponding verticals of it.

About the only advantage that can be claimed

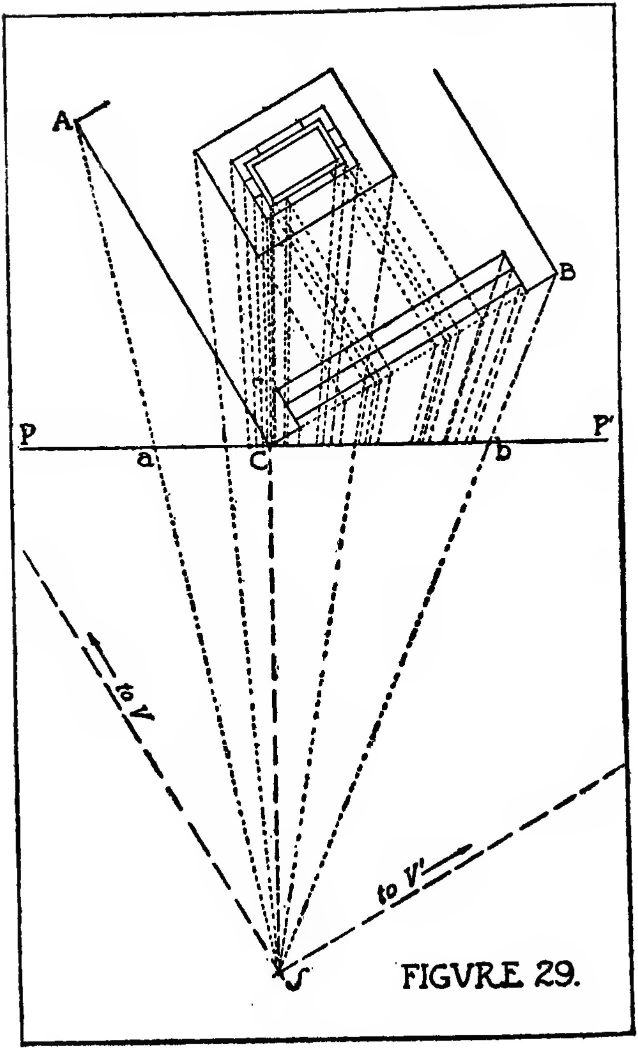


FIGURE 29.

for this method of direct projection is that often we already have drawn out an orthographic plan which may be used in laying out our perspective and save us the drawing of the necessary perspective plan used in the other system. It has many disadvantages, however, to offset this: it takes much more working space for the orthographic plan, with subsequent projection of points on *PP′*, than it does for the perspective plan (see Figure 28); there always results a confusing array of points on *PP′*, very often leading to error, while the perspective plan is complete in itself and may be made on tracing paper over the lower part of the drawing and directly below the picture, thus being removable and easily filed for future reference. In the method of direct projection it is usually impracticable to draw the perspective picture directly under or over the plan-diagram, necessitating the transfer of the projected points on *PP′* to the picture by strip of paper or scale, another very possible source of error. In the method of perspective plan, however, as many perspective plans of the different parts as are necessitated by the complexity of the subject may be drawn either above, below or through the perspective itself. The perspective plan is also easily changed; in fact, a design may be studied and modified directly in perspective with only the crudest and most meager preliminary sketches for a guide. Altogether, the method of perspective plan is the more scientific, more compact and clean-cut method of the two, and is much preferred by up-to-date draftsmen.

This method of direct projection on the picture plane can be further extended to drawing a perspective without recourse to vanishing points. In the center of Figure 30 is shown the plan and projection to point of station of a rectangular block with the vertical edges located by projected points *b, a, d* on *PP′* in a manner similar to that of Figure 28. At the left of Figure 30 is the side elevation of the block taken so that the line *pp′* is the side elevation of the picture plane of which *PP′* is the plan, and *S* is the side elevation of the point of station *SG* high above the ground *G*. If we project lines from *A, B, D, A′, B′, D′*, of the left-hand figure to S, the intersections of these lines with *pp′, a, b, d, A″, B″, D″*, will determine the perspective lengths of the vertical edges of block *AA′, BB′* . . . and also their distance below the horizon *HH* or above the ground line *GG* on the picture plane of the central perspective. Thus by laying off these distances on the corresponding verticals drawn under the determining points of plan (*baCd*), in central figure, we may draw the perspective of the rectangular block without having used the vanishing points.

The right-hand side of Figure 30 shows how the two diagrams of the central and left-hand figures may be combined. Through *S* we draw a line parallel to *PP′*, and on this line locate *G*, *SG* distant from *S* and on either side of *S*. We

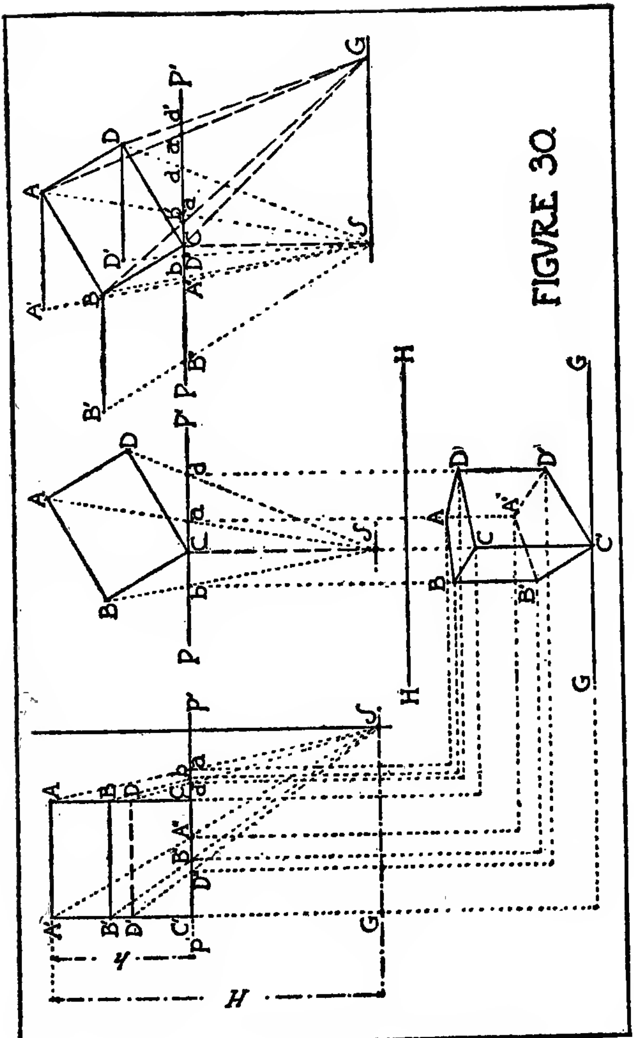


FIGURE 30.

now project lines from the points A, B, C, D to G, intersecting PP' in a', b', C, d'. The distances aa', bb', dd', are equal to $C'A''$, $C'B''$, $C'D''$ of the left-hand figure, or equal to the distances of A', B', and D' above the ground line in the central figure.

This may be easily proven by plane geometry. In the triangles $A'GS$ of the left-hand figure and ASG of the right-hand figure, the perpendicular distance of A' from GS and of A from SG are both equal to H. The perpendicular distance of A' from $C'A''$, which is parallel to GS of the left-hand figure, and of A from aa', which is parallel to SG of the right-hand figure, are both equal to h. In the left-hand figure, the triangles $A'GS$ and $A'C'A''$ are similar, hence the area of $A'GS$ is to the area of $A'C'A''$ as H is to h. In the right-hand figure, the triangles ASG and Aaa' are similar, hence their areas are to each other as H is to h; but the area of ASG, $1/2H \times SG$, is equal to the area of $A'GS$, $1/2H \times GS$, therefore the area of $A'C'A''$, $H/h \times A'GS$ is equal to the area of Aaa', $H/h \times ASG$. But the altitudes of $A'C'A''$ and Aaa' are equal, h; then, since their areas are equal, their bases, $C'A''$ and aa', must be equal. Likewise bb' and dd' are respectively equal to $C'B''$ and $C'D''$.

If through A, D and B of the right-hand figure we draw lines parallel to PP' and on these lines locate A', D' and B' so that AA', DD' and BB' will correspond to similarly lettered lines of the left-hand figure; or, in other words, if we lay off on these lines the lengths of the verticals at these points, and from A', D', B' project lines to S, intersecting PP' at A'', D'' and B'', then the lines $A''a$, $D''d$, $B''b$ will be the perspective lengths of the verticals and equal respectively to $A''a$, $D''d$ and $B''b$ of the left-hand figure. The correctness of this is proven in the same way that we proved similar distances equal in the last paragraph. The combined diagram on the right-hand side of Figure 30 gives all points and lines necessary for the completion of the perspective without recourse to vanishing points.

Another method of drawing a perspective without using the ordinary vanishing points is shown in Figures 31 and 31*A*. This is constructed by means of ordinates and parallel or one-point perspective. From the points B'', A'' and D'' of Figure 31*A*, the lines (ordinates) $B''b''$, $A''a''$ and $D''d''$ are drawn perpendicularly to PP'. These lines, being perpendicular to PP', may be drawn in parallel perspective to V as shown in Figure 31 by $b''B'$, $a''A'$ and $d''D'$. At a convenient point on the ground line, as c', Figure 31, we draw a parallel perspective of the oblique side elevation, $c'caa'$. We now project lines from d', b', a' to D', B', A'. We connect the points D', B', A' and draw verticals on them; by projecting from c, d, b and a, we find C, D, B and A on these verticals, and by connecting these points complete the perspective. This method is more compact than the one of Figure

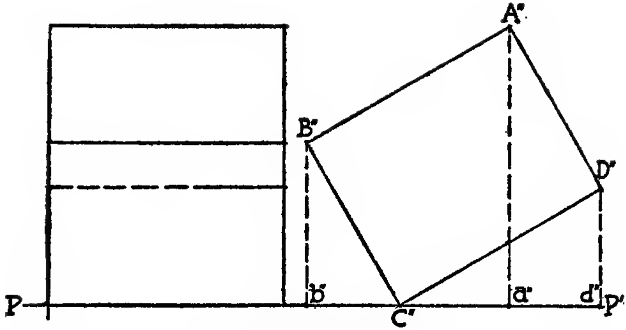


FIGURE 31A.

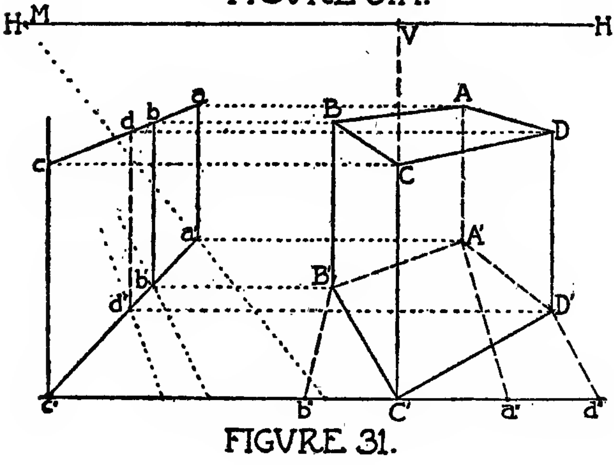


FIGURE 31.

30, and if fractional measuring points are used in drawing the parallel perspective as explained in the last chapter, it may be worked in very little space.

In Figure 32 is shown another very convenient way of getting the perspective length of verticals or perspective heights. Having drawn the

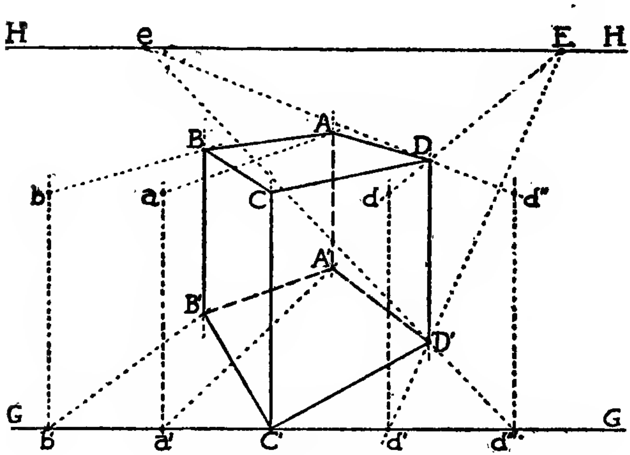


FIGURE 32.

perspective ground plan, *A′B′C′D′*, by any of the methods outlined above, we draw indefinite verticals at the points *A′*, *B′*, *C′*, *D′*. We now take any convenient point, *E*, on the horizon line, *HH*, and from *E* we draw a line through *D′* and find its intersection with the ground line, *GG*, at *d′*. If on a vertical drawn at *d′* we place *d* so that *dd′* is equal to the actual height of *DD′* and from *d* draw a line to *E*, the intersection of *dE* and the vertical at *D′* will give us *D*. Likewise, as shown by lines drawn from *a′* and *a*, *b′* and *b*, to *E*, we may find *A* and *B*. We may assume any other point on *HH* as *e*, and draw from *e* through *D′* to *d‴* on *GG*, then locate *d″* on a vertical at *d‴* so that *d″d‴* is equal to the actual height of *DD′*, and by drawing from *d″* to *e* obtain the same intersection on the vertical at *D′*, *D*, as heretofore. The reason for this is that *E* on the horizon line is the vanishing point for the line *d′E*, *d′* is the intersection of this line with the vertical picture plane, and a vertical at *d′* must be a front line and a line of measures or vertical scale for any vertical in the vertical plane on *d′E*, or of which *d′E* is the horizontal trace. Similarly, this is true for *e* or any point on the horizon line. (See Chapter VI.)

Although Figures 30, 31 and 32 are the perspectives of simple rectangular blocks, if the principles used in making these are thoroughly understood, any objects may be drawn by these methods. We must repeat that the solution of any problem in perspective may be resolved into finding the perspectives of a series of points; for any straight line is determined by two points on it, and any line may be determined by a series of points on it, and objects are pictured by their defining lines or outlines. Still again, we must repeat that the variety and readiness of application of a few primary principles rather than a variety of principles are the essential things in mastering perspective.

The methods above given for laying out perspectives without recourse to vanishing points, although of no very great importance to the architectural draftsman, are often useful and convenient. In interior perspectives, where the furniture is placed at various angles with the walls, it would necessitate the finding of a set of vanishing points for each piece of furniture as well as a set for the room itself, while by some such method as given above the furniture might easily be laid out without the use of vanishing points. In very large drawings or in mural paintings, where it is usually impracticable and sometimes impossible to utilize ordinary vanishing points, these methods or similar ones may be used to advantage. Often the simple expedient is utilized of making the drawing at a comparatively small scale and then proportionately enlarging it to fit the larger panel. This is sometimes done by assuming a supplementary picture plane in front of the actual picture plane, where the image is consequently at a reduced scale, and then projecting this reduced image to the actual picture plane at the normal scale. There are several other schemes for accomplishing the same results, as well as several ingenious developments and combinations of these various methods by which a perspective drawing may be laid out practically within the limits of the picture itself, for a more detailed explanation of which the student is referred to the more comprehensive treatises on the subject, such as "Modern Perspective."

X

OBLIQUE AND INCLINED LINES AND PLANES

AS a preliminary to the more careful consideration of oblique and inclined lines and planes, Chapters V and VI should be thoroughly understood and a careful rereading of these is advised.

By oblique lines are meant lines which make oblique angles with the picture plane but are parallel to the ground plane. By inclined lines are meant lines which make oblique angles with the ground plane. By oblique planes are meant planes which make oblique angles with the picture plane but are perpendicular to the ground plane. By inclined planes are meant planes which make oblique angles with the ground plane. In an oblique perspective of a gabled house, the lines of the gable are inclined. The ridge and eaves lines are oblique, being parallel to the ground plane. The plane of the gables is oblique, being perpendicular to the horizontal ground plane. The roof slope plane is inclined.

In Chapter V, Figures 12, 12*A*, 12*B*, is given a simple method of finding the vanishing points of inclined lines. There is a more scientific method of much broader application for finding these when the angle of inclination with the horizontal ground plane is given or may be found. This is shown in Figure 33.

The gable lines, 1–2, 3–4, 2–3, 4–5, are inclined at the angle of 30°. If we draw a vertical through *V′*, a line drawn through *M*, the measuring point for horizontal lines in the plane of the gable lines, at 30° to *HH*, will intersect this vertical at *V′d*, the vanishing point of 1–2 and 3–4. A similar line drawn below the horizon will give *V′d′*, the vanishing point of 2–3 and 4–5.

To get the vanishing point of an inclined line, then, we draw a vertical through the vanishing point of a horizontal line in the same plane as the inclined line, and from the measuring point for this horizontal line, draw a line at the same angle with the horizon as the angle of inclination. The intersection is the vanishing point required. The explanation of this is simple.

In Figure 33, the line *V′d–V′–V′d′* is the vanishing line of the plane of the gables, 1, 2, 3, 4, 5, and planes parallel to it. (See Chapter V.) The plane *S*, *V′*, *V′d*, *V′d′* is parallel to the plane of the gables, 1, 2, 3, 4, 5. A line, in this plane, from *S* and at 30° to the horizontal *S–V′*, will be, above the horizon, parallel to 1–2 and 3–4 and, below the horizon, to 2–3 and 4–5. The intersections of these lines from *S* with *V′d–V′–V′d′*,

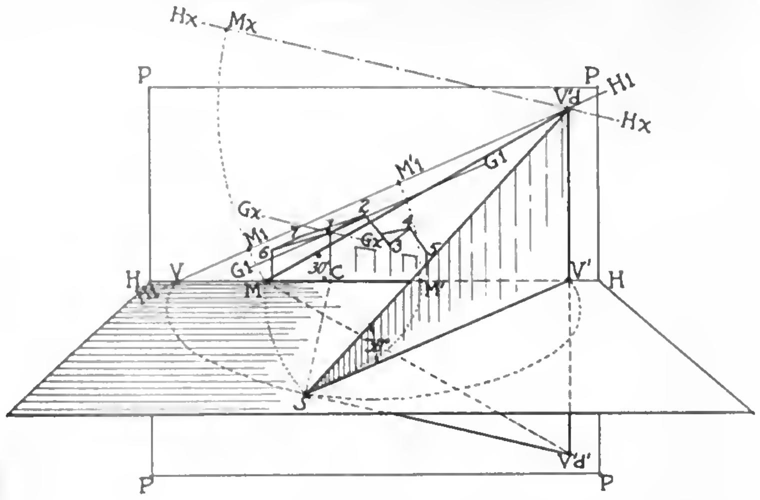


FIGURE 33

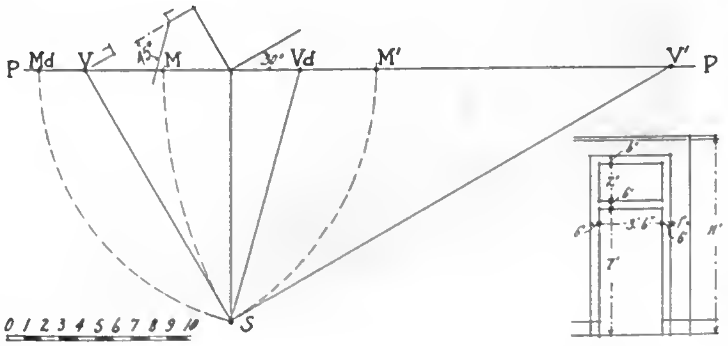


FIGURE 34A

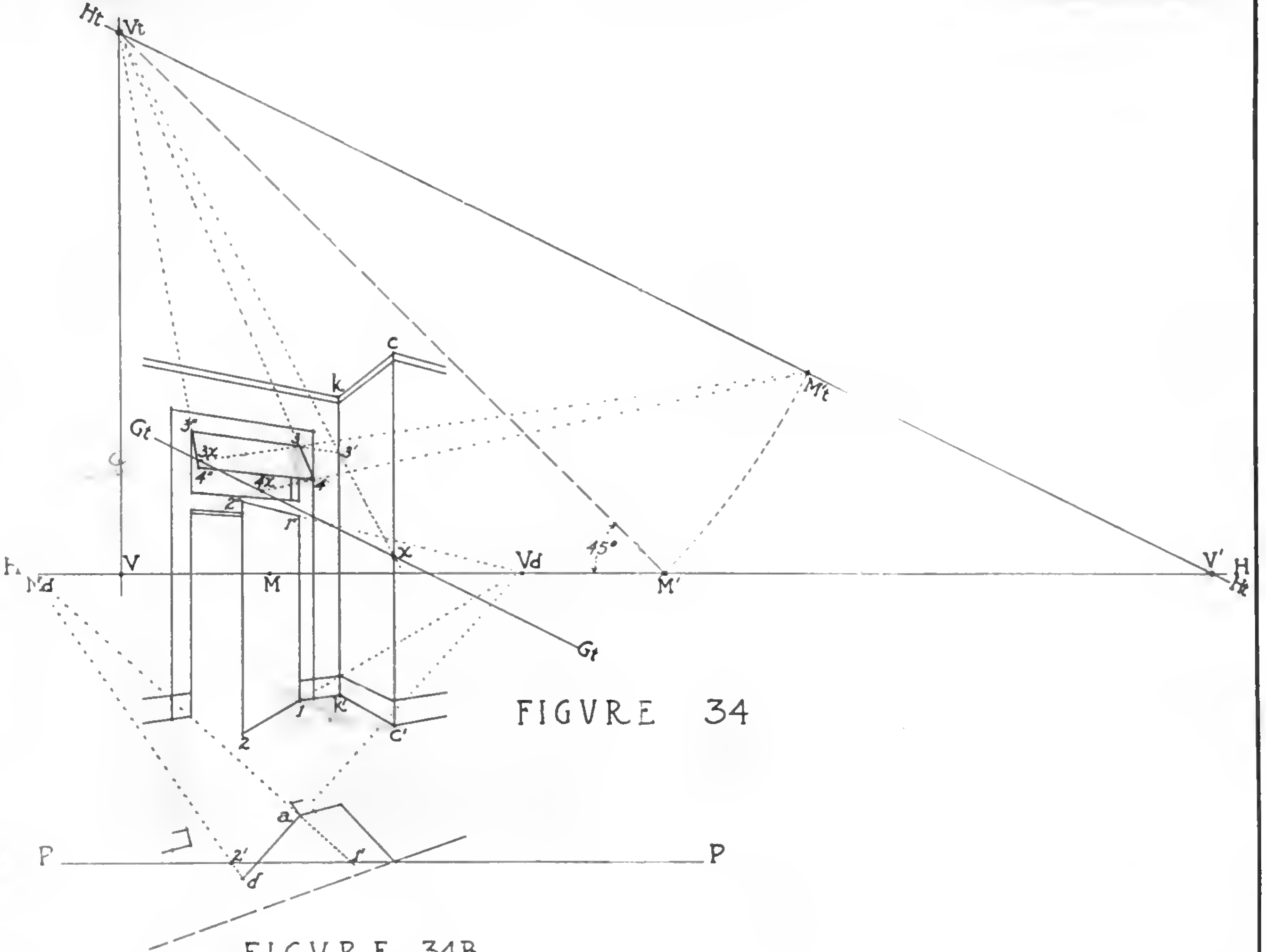


FIGURE 34

FIGURE 34B

the vanishing line of their plane, will give $V'd$ and $V'd'$, the vanishing points of these lines and their parallels, the gable lines. But the distance, $V'S$, from V', the vanishing point of the horizontal line, to the point of station, S, is equal to $V'M$, the distance from the vanishing point to the measuring point of the line. This, as is shown in Chapter VI, is always true. If we revolve the triangles $V'd,V',S$ and $V'd',V',S$ on the axis, $V'd$–V'–$V'd'$, into the picture plane, the line V'–S will coincide with the line V'–M, the point S will coincide with the point M and the angle at S with the angle at M, for they are each 30°. Therefore the triangles, $V'd,V',M$ and $V'd,V',S$ will coincide, as also will the triangles, $V'd',V',M$ and $V'd',V',S$. The line S–$V'd$ will coincide with M–$V'd$ and the line S–$V'd'$ with M–$V'd'$. The construction for $V'd$ or $V'd'$ by drawing the inclined lines from M is therefore exactly equivalent to actually drawing these inclined lines from S.

We will now consider the inclined roof plane 1, 2, 6, 7. The vanishing point of the lines 1–2 and 6–7 is $V'd$. The vanishing point of the lines 1–6 and 2–7 is V. A line, $H1$–$H1$, drawn through V and $V'd$ is the vanishing line or the horizon line of the plane, 1, 2, 6, 7, and planes parallel to it. (See Chapter V.) Since the measuring point for distances on a line is always the same distance from the vanishing point of the line as is the point of station, we may find $M1$, the measuring point for 1–2, by laying off $M1$–$V'd$ equal to *S–V'd* or *M–V'd*, and we may find *M'*1, the measuring point for 1–6, by laying off *M'*1–*V* equal to *S–V* or *M'–V*.

Since point 1 is in the picture plane, a line, *G*1–*G*1, drawn through it and parallel to *H*1–*H*1, the horizon line, which is also in the picture plane, will be in the picture plane (any two parallel lines lie in the same plane), and therefore this line, *G*1–*G*1, is a front line and a measuring line. On this line we may lay off actual scaled distances and foreshorten them to 1–2 by the measuring point *M*1. This will all look quite natural if we rotate the page until *H*1–*H*1 becomes horizontal. The lines and points will then appear in their more familiar attitudes.

The horizon line is the projection on the picture plane of the actual horizon or vanishing line of a plane. As an infinite number of planes may pass through a single line, a line has an infinite number of horizons passing through its vanishing point. We usually use, for the sake of convenience, that horizon which passes through one or more other vanishing points of related lines in the same or parallel planes. In Figure 33, we use for 1–2 and 3–4, the horizon *H*1–*H*1, which is the vanishing line for the roof plane and its parallels and contains all vanishing points and measuring points for lines in this plane or its parallels. For measurements on 1–2, however, we might have drawn any horizon line through *V'd*, its vanishing point, such as *Hx–Hx*, find *Mx* by making *Mx–V'd* equal to *V'd–S* or

V′d–M, and lay off scaled distances on a line *Gx–Gx* through 1 and parallel to *Hx–Hx*.

All this is just as true for one point or parallel perspective, where one system of lines is parallel to the picture plane and has an infinite vanishing point and the other system is oblique to the picture plane and has a finite, accessible vanishing point and measuring point.

The pages just gone over must be thoroughly understood before we may go on to the fascinating work in inclined lines and planes and the perspective of shadows. As was said in the beginning, a sure comprehension of Chapters V and VI is a necessary preliminary.

Figure 34 shows a problem involving the perspective drawing of inclined lines and planes. The drawing of the walls and the door-frame is done in the usual way. The unfamiliar points occur in the drawing of the transom, hinged at its top and swung out at an angle of 45° with the plane of the wall; and of the door, which is hung at the right and swung open at an angle of 45°.

The diagram for the construction of the various points, Figure 34*A*, is drawn similarly to the diagrams in Figures 7 and 10*A*. We find *Vd*, the vanishing point for the horizontal lines of the door, simply by drawing a line from *S*, the point of station, parallel to the door line and intersecting the line of the picture plane, *P–P*, at *Vd*. We find the measuring point for the door line, *Md*, by laying off *Md–Vd* equal to *Vd–S*, in the usual way; the measuring point *Md* being the same distance from *Vd*, the vanishing point, as *Vd* is from *S*, the point of station.

The perspective plan, Figure 34*B*, is laid out similarly to others in previous problems except in so far as the door line is concerned. For this we first draw a line through *a* from *Vd*. We next locate 1′ by drawing a line from *Md* through *a*. 1′ is the actual point on the line of measures *P–P*, corresponding to the perspective, *a*. This is the reverse of the usual process when we take an actual point on the line of measures and find its perspective by means of the measuring point. Here we take the perspective of a point and by inverse projection from the measuring point, find its actual location on the line of measures. We now lay off the width of the door, 3½ feet at the proper scale, on *P–P* from 1′ to 2′. We draw a line from *Md* through 2′ and find *d*, the perspective of 2′. The line *a–d* is the perspective plan of the door.

In Figure 34, we draw lines from *Vd* through 1 and 1″, and on these lines locate 2 and 2″ over *d* of plan. 1, 2, 2″, 1″ is the perspective of the door.

To draw the transom, we must first find the vanishing point of the line 3–4, which is inclined at 45° with the horizontal plane. We do this as we did in Figure 33. Since the line 3–4 is parallel to the plane *c*, *c′*, *k′*, *k*, its vanishing point must lie in the vertical line through *V*, this vertical line being the vanishing line of the plane *c*, *c′*, *k′*, *k* and its parallels. We next draw a line from *M′* at 45° (the inclination of 3–4), to *H–H* and intersecting *V–Vt* at *Vt*, the vanishing point of 3–4. We now draw lines of indeterminate length from *Vt*, through 3 and 3″.

Since one set of lines in the plane of the transom vanish in *V′*, another set in *Vt*, a line drawn through *Vt* and *V′*, *Ht–Ht*, is the horizon line for the transom plane. On this line, we lay off *M′t–Vt* equal to *M′–Vt*, finding the measuring point, *M′t*, for 3–4 and its parallels.

We now continue 3–4, or one of its parallels, to its intersection with the picture plane. Through this intersection, we can then draw a front line and a line of measures. One way to do this is to continue 3″–3 to 3′, its intersection with *k–k′*. Through 3′, we now draw a line from *Vt* to *x*, its intersection with *c–c′*. Since *c–c′* is a front line lying in the picture plane (drawn so), *x* lies in the picture plane. We draw *Gt–Gt* through *x* and parallel to *Ht–Ht*. *Gt–Gt* is a front line and a line of measures for 3′–*x* or its parallel, 3–4. We now find 3*x* on *Gt–Gt* by drawing a line from *M′t* through 3. We lay off 3*x*–4*x*, two feet long at the given scale, and project 4 on 3–4 by drawing from *M′t* through 4*x*. 4–4″, drawn to *V′*, completes the transom. We might have just as easily projected 3″ to *Gt–Gt*, scaled the distance for the width of transom and found 4″.

This problem and similar ones with varying angles should be practised until the principles and manipulations have become quite familiar. Mere studying of the text and illustrations is not enough. The figures must be redrawn following the text and then without the assistance of the text. Finally, variations of the problem should be drawn.

XI

THE PERSPECTIVE OF SHADOWS

BY SUNLIGHT

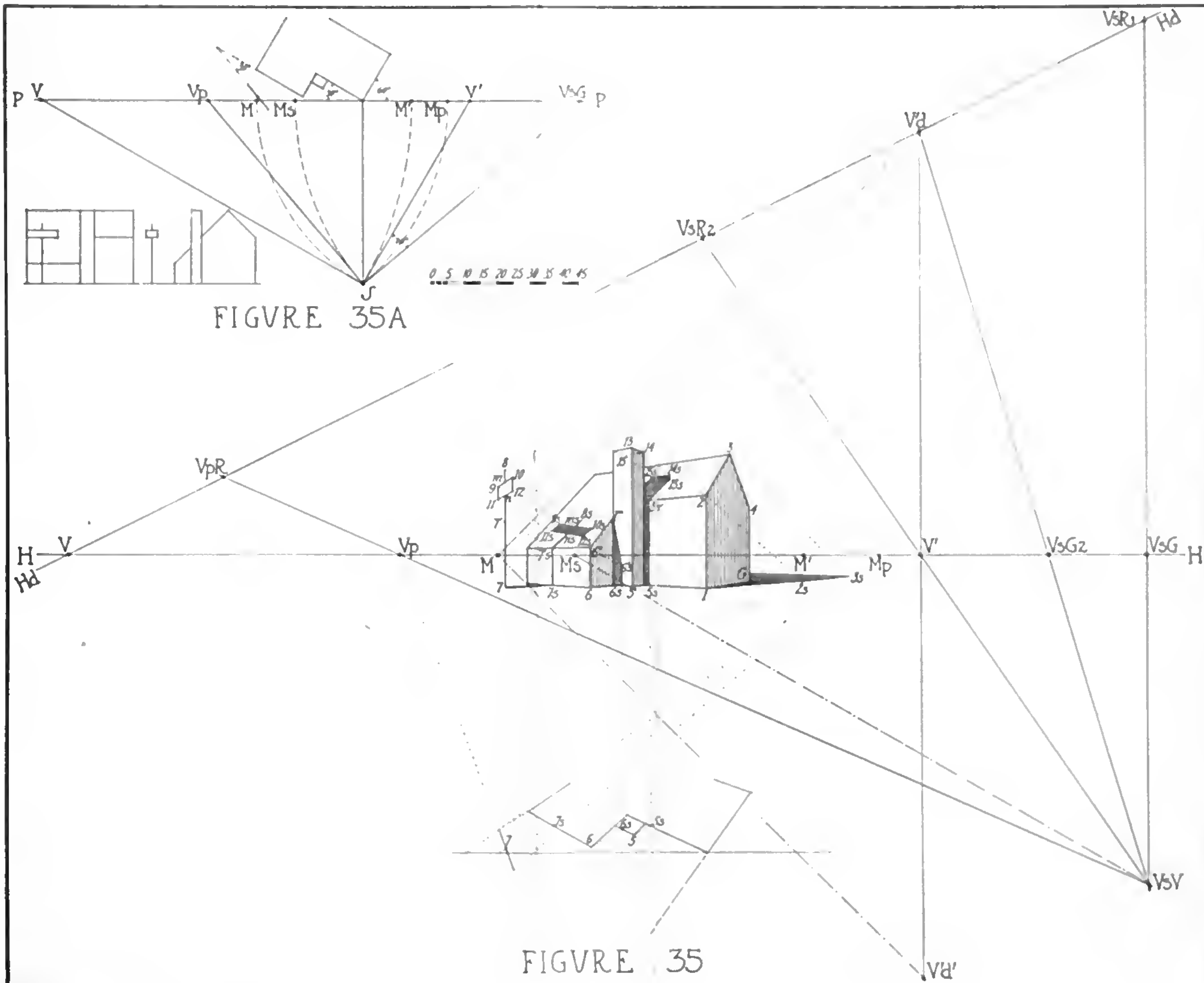
IN drawing shadows cast by sunlight, the sun is assumed to be an infinite distance away, hence its rays of light are parallel. The shadow of a point on any surface is the intersection with the surface of the ray of light passing through the point. The shadow of a straight line on any surface is the intersection with the surface of the light plane generated by the light rays passing through the points of the straight line. The shadow of any line on any surface is the intersection with the surface, of the light surface generated by the light rays passing through the line.

Although we call the lines of light which pass through a point, light rays, and the surfaces generated by these light rays passing through the points of a line, light planes and light surfaces, we must remember that these lines and surfaces are light only from the source of light to the points or lines through which they pass. From these points or lines to the surface on which the shadow is cast, they are lines and surfaces of darkness.

In the case of a straight line, if we know the direction of its shadow on a plane and can find the shadow of one of its points, we may draw the shadow through the one point known. In the case of the shadow of a limited straight line on a plane, which shadow is then a limited straight line, we may draw the complete shadow by connecting with a straight line the shadows of the points of extremity of the line. We can plot the shadow of any line on any surface by finding the shadows of points close together on the line. If then, we can cast the shadow of any point on any surface, we can cast the shadow of any line on any surface and then we can cast the shadow of any surface bound by lines or any solid bound by surfaces, on any surface.

Figure 35*A* is a plan and elevations diagram of a barn and sign post of which Figure 35 is the perspective. To avoid the conventional angle of 45°, we assume that the sun's rays make an angle of 20° with the ground line of the narrow side of the barn and an angle of 30° with the horizontal. The sign board makes an angle of 20° with the long side of the barn. The roof slope is 45°.

In Figure 35*A*, *Vp*, the vanishing point for the horizontal lines of the sign board, and *VsG*, the vanishing point for the ground lines or projections on the ground plane, of the light rays, are found in the usual way by drawing lines from *S*,



FIGVRE 35A

FIGVRE 35

the point of station, parallel to these lines on the plan. *Mp,* the measuring point for the horizontal lines of the sign board, is found in the usual way by laying off *Mp–Vp* equal to *S–Vp*. *Ms* is similarly found by making *Ms–VsG* equal to *S–VsG*. These points are then transferred to Figure 35.

In Figure 35, *V′d* and *V′d′*, the vanishing points for the roof slope lines, are found as explained in Figure 33, by drawing from *M*, 45° lines to the vertical through *V′* and intersecting the vertical, in *V′d* and *V′d′*. The perspective of the barn and sign post may now be drawn, when we are ready to cast the shadows.

We shall begin with the shadow of the building on the ground at the right. First we must find *VsV*, the vanishing point of the sun's rays, lines which make an angle of 30° with the horizontal and whose horizontal projections make an angle of 20° with the ground line, 1–*G*. We have already found *VsG*, the vanishing point of these horizontal projections of the light rays. It is quite obvious that 1–2*s*, the shadow on the ground of the vertical, 1–2, is the intersection with the ground plane of a plane generated by the light rays passing through the points of the vertical, 1–2. It is also quite obvious that 1–2*s* makes the assumed angle of 20° with 1–*G* and vanishes in *VsG*. The vertical through *VsG* is the vanishing line of the vertical plane generated by the light rays passing through 1–2 and in which plane lies 2–2*s*, a light ray making the assumed angle of 30° with the horizontal. If then we draw from *Ms,* a line below the horizon and making an angle of 30° with it, it will intersect the vertical through *VsG* at *VsV*, the vanishing point of 2–2*s* and its parallels, the other light rays. This is simply a problem in inclined lines.

If we draw the line 2–2*s* from 2 to *VsV* to its intersection with 1–2*s* at 2*s*, 2*s* will be the shadow of 2, and 1–2*s* the shadow on the ground of the vertical 1–2.

The shadow of 2–3 on the ground, which is 2*s*–3*s*, is the intersection of a plane generated by the light rays passing through 2–3, the roof line. Since 2–3 lies in this light plane, its vanishing point, *V′d,* must lie in the vanishing line of the light plane. The light rays, 2–2*s* and 3–3*s*, lie in this same plane, hence their vanishing point, *VsV*, must lie in the vanishing line of the plane, and the line drawn through *VsV* and *V′d,* two points on the vanishing line, must be the vanishing line of the light plane. The line 2*s*–3*s* is a horizontal line (on the ground), hence its vanishing point must lie in the horizon line *H–H.* The line 2*s*–3*s* also lies in the light plane passing through 2–3 and whose vanishing line, *V′d–VsV*, we have just found. The vanishing point of 2*s*–3*s* must also lie, therefore, on *V′d–VsV* as well as on *H–H,* and must be the intersection of *V′d–VsV* and *H–H* or *VsG*2. This is one of the most interesting manipulations in perspective drawing and occurs repeatedly in the casting of shadows. It should be thoroughly understood.

Having found *VsG*2, we draw 2*s*–3*s* to it from 2*s*. A line drawn through 3 to *VsV* intersects 2*s*–3*s* at 3*s*, the shadow of 3. A line drawn from 3*s* to *V* gives the shadow of the ridge line and completes the shadow of the building at the right. The shadow of the ridge line is drawn to *V* because it is the intersection with the ground plane of the light plane generated by the light rays passing through the ridge line, which is parallel to the ground plane. The intersection or shadow is therefore parallel to the ridge line and drawn to the same vanishing point, *V*. Since the line 3–4 falls within the line 3–3*s*, the shadow of 3–4 falls within the shadow of the ridge line and does not show.

By drawing from 5 to *VsG*, we get 5*s* and a vertical at 5*s* gives 5*s′* at the eaves. The point 5*s* (as also the points 6*s* and 7*s*) may be found more easily and accurately on the perspective plan, as shown. This determines the shadow of the chimney on the ground and on the side wall of the barn. We must next find the shadow of the line 5–13 on the roof, 5*s′*–13*s*. This line, 5*s′*–13*s*, is the intersection with the roof plane, of a light plane generated by the light rays passing through the points of 5–13, hence the vanishing point of this intersection, 5*s′*–13*s*, lies in the vanishing line of the light plane, *VsV*–*VsG* continued, and also in the vanishing line of the roof plane, *Hd*–*Hd*, and must be the intersection of these two lines at *VsR*1. We therefore draw 5*s*′–13*s* to *VsR*1, and a line from 13 to *VsV* determines 13*s*, the shadow of 13.

The shadow of 13–14 on the roof is the intersection with the roof of a light plane generated by the light rays passing through 13–14. The vanishing line for the light plane passing through 13–14, is the line passing through *V*′, the vanishing point of 13–14, and *VsV*, the vanishing point of the light rays. This line continued intersects *Hd–Hd*, the vanishing line of the roof plane, in *VsR*2, the vanishing point of 13*s*–14*s*. A line from 14 to *VsV* determines 14*s*, the shadow of 14.

A line drawn from 14*s* to *V* gives the line of the shadow of 14–15. The shadow of 14–15 on the roof plane, 14*s*–15*s*, is the intersection with the roof plane of a light plane passing through 14–15. Since 14–15 is parallel to the roof plane, the intersection or shadow, 14*s*–15*s*, is parallel to it and goes to *V*, the same vanishing point. A line from 15 to *VsV* gives 15*s*, the shadow of 15.

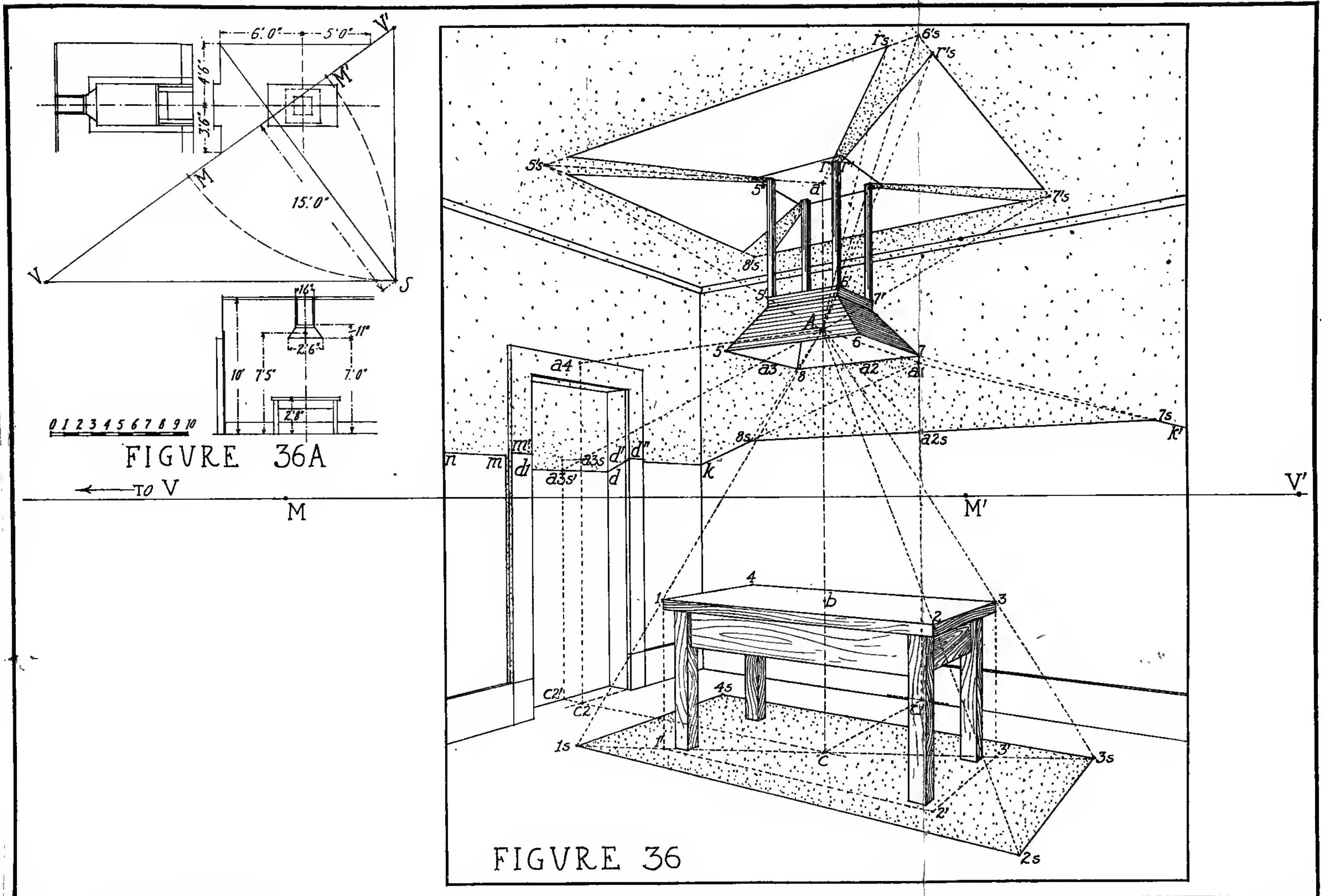
A line from *VsR*1 through 15*s* completes the shadow of the chimney on the roof. This shadow line is obviously parallel to 5*s*′–13*s*, being the shadow of a line parallel to the one of which 5*s*′–13*s* is the shadow and cast on the same plane. It therefore has the same vanishing point as 5*s*′–13*s*, *VsR*1.

A line from 6 to *VsG*, intersecting the ground line of the chimney at 6*s*, where we draw a vertical on the face of the chimney, will give us the intersection with the ground and chimney of the light plane passing through 6–6′. A line from 6′ to *VsV*, gives 6′*s*, the shadow of 6′ on the chimney. By connecting *r* and 6′*s*, we complete the shadow of the projecting wing on the ground and chimney. The line *r*–6′*s* may be drawn by finding its vanishing point as others were found —by intersecting vanishing lines—but this vanishing point is inaccessible and in the present instance entirely unnecessary.

A line from 7 to *VsG* gives 7*s* at the ground line of the extension or wing of the barn, 7–7*s* being the shadow of the sign post on the ground. A vertical at 7*s* to 7′*s*, gives 7*s*–7′*s*, the shadow of the sign post on the wall. By drawing from *VsV*, through 7′*s* to 7′ on the sign post, we find that 7′*s* is the shadow of 7′. Finding 7′ is unnecessary but interesting. A line from 7′*s* to *VsR*1 gives the shadow of the sign post on the roof, and a line to *VsV* from 8 gives 8*s*, the shadow of 8, the extremity of the sign post, on the roof. Lines from *m* and *n* to *VsV* give *ms* and *ns*, the shadows of *m* and *n* on the roof. The shadow of the line 9–10 on the roof, 9*s*–10*s*, is the intersection with the roof plane of a light plane generated by light rays passing through the line 9–10. The vanishing line of this light plane passes through *Vp*, the vanishing point of the line 9–10, one of its elements, and through *VsV*, the vanishing point of the light rays, also elements of the same plane. The vanishing point of the line 9*s*–10*s*, which lies in the vanishing line of the light plane, *VsV–Vp*, and also lies in the vanishing line of the roof plane, *Hd–Hd*, must be the intersection of these two vanishing lines, or *VpR*. The line 11*s*–12*s*, which is the shadow on the same roof plane, of the line 11–12, a parallel to the line 9–10, is parallel to 9*s*–10*s*, the shadow of 9–10, and hence goes to the same vanishing point, *VpR*. We, therefore, draw lines through *ms* and *ns* to *VpR* and on these lines locate 9*s*, 10*s*, 11*s* and 12*s*, the shadows of 9, 10, 11 and 12, by drawing from these points, 9, 10, 11 and 12, to *VsV*. We check this by finding that the lines 9*s*–11*s* and 10*s*–12*s* go to *VsR*1, the vanishing point for shadows, on the roof, of vertical lines.

This completes this drawing, which involves every basic principle of the perspective of shadows.

The student should redraw this several times, first by following carefully the instructions given and then with variations without the instructions.



FIGVRE 36A

FIGVRE 36

To face p 95—Lubschez "Perspective"

XII

THE PERSPECTIVE OF SHADOWS

BY ARTIFICIAL LIGHT

THE general theory of shadows by artificial light is similar to that of shadows by sunlight. With shadows by artificial light, however, the rays of light are not parallel but divergent, radiating from a point at the center of the source of light.

As with sunlight, the shadow of a point on any surface is the intersection of the light ray passing through the point with the surface on which the shadow is cast. The shadow of a line on any surface is likewise the intersection of the surface generated by the light rays passing through the points of the line, with the surface on which the shadow is cast.

Light rays, instead of being drawn to a vanishing point, are drawn to the point at the center of the source of light.

In Figure 36, which is the perspective of an interior with table and lamp and of which Figure 36*A* shows the plan and elevations, it is required to show all the shadows on the walls, ceiling and floor cast by the light at *A*.

The drawing of the room and accessories should not present any difficulties. We may use the floor itself for the perspective plan as we did in the parallel or one point perspective in Figure 18*B*.

To find the shadow of the table top on the floor, we must find where lines passing through the source of light, *A*, and the corners of the table top, 1, 2, 3, 4, pierce or intersect the floor at 1*s*, 2*s*, 3*s*, 4*s*. The line *A–b–c* is drawn perpendicular to the floor and *c* is the projection of *A*, the source of light, on the floor, while *a* is its projection on the ceiling. The point *c* is the perspective plan, on the floor of *A*. In fact *c* is drawn first and *A* is located at the proper height on a vertical at c. It is quite obvious that any plane passing through *A–c* will be perpendicular to the floor. Since a plane may be drawn through any two intersecting lines, a plane, *A,c,*1*s,* may be passed through *A–c* and *A*–1–1*s*. The plane *A,c,*1*s* must necessarily pass through *c*–1*s*, its intersection with the floor. Since 1′ is the projection, on the floor, of 1, the line 1–1′ is perpendicular to the floor at 1′ and lies in the plane *A,c,*1*s*, and 1′ lies in the intersection of the plane *A,c,*1*s* with the floor, the line *c*–1*s*.

Therefore to find 1*s*, we draw a line, *c*–1′ continued, from *c* through 1′ and another line, *A*–1 continued, the light ray from *A* through 1. The intersection of *c*–1′ continued and *A*–1 continued gives 1*s*, the shadow of 1.

We might proceed in the same way we found 1*s*, to find 2*s*, 3*s* and 4*s* and connect these points for the shadow of the table top, but this is unnecessary. The line 1*s*–2*s*, the shadow of the line 1–2, is the intersection of the light plane generated by the light rays passing through the line 1–2, with the floor and is parallel to the line 1–2 since this line is parallel to the floor. We therefore simply draw a line through 1*s* to *V*, the vanishing point of 1–2 and its parallels. By drawing a line from *A*, the source of light, through 2, we find the intersection 2*s*, the shadow of 2. Similarly the line 2*s*–3*s*, the shadow of 2–3, is parallel to 2–3 and is drawn from 2*s* to *V′*, the vanishing point of 2–3 and its parallels. The intersection, 3*s*, the shadow of 3, is located by drawing from *A* through 3. Just as we did before, we draw 3*s*–4*s* from 3*s* to *V* and draw 1*s*–4*s* from 1*s* to *V′* and thus complete the shadow of the table top.

A similar process is employed to find the shadow of the lower edge of the lamp shade on the walls. Draw a perpendicular from *A* to the back wall at *a*1. This is done by drawing from *c* to *V′*, intersecting the ground line of the wall at *c*1, then drawing from *A* to *V′*, intersecting this vertical at *a*1. It is plain that a plane passing through the lines *A*–*a*1 and *a*1–*c*1 will be perpendicular to the wall and to the lower edge of the lamp shade at its middle point, *a*2. The light ray *A*–*a*2 will lie in this plane and intersect *a*1–*c*1 at *a*2*s*, the shadow of *a*2. The shadow of the line 7–8 is the intersection with the wall of a light plane generated by the light rays passing through 7–8. This intersection is parallel to the line 7–8 since 7–8 is parallel to the wall, and may be drawn through *a*2*s* to *V*, the vanishing point for 7–8 and its parallels. By drawing lines from *A* through 7 and 8, the intersections on this line through *a*2*s*, 7*s* and 8*s*, may be found; these are the shadows of 7 and 8 and the line 7*s*–*a*2*s*–8*s* is the shadow of the line 7–8.

The lines 8*s*–*k* and 7*s*–*k′*, the shadows on the back wall of parts of the lines 8–5 and 7–6, are the intersections of the light planes generated by the light rays passing through 8–5 and 7–6. These light planes are parts of unlimited planes passing through *A* and through 8–5 and 7–6. These unlimited planes intersect the wall in *a*1–8*s*–*k* and *a*1–7*s*–*k′*. The shadows 8*s*–*k* and 7*s*–*k′* may therefore be drawn by drawing through 8*s* and 7*s*, lines from *a*1, the projection of *A*, the source of light, on the back wall.

The point *k* in the corner, and therefore in both wall planes, is the shadow of a point on the line 8–5. The shadow of 8–5 on the side wall is parallel to 8–5 since 8–5 is parallel to the side wall, therefore a line drawn through *k* to *V′* gives *k*–*d′*–*a*3*s*–*m*–*n*, the shadow of 8–5 on the side wall. The same construction that we used to find 7*s*–8*s* may be used to find *k*–*d′*–*a*3*s*–*m*–*n* and is also shown. In fact this construction is necessary to find *d*–*d*1, the deeper shadow on the door plane—*d*–*d*1 is drawn through *a*3*s′* to *V′*.

By continuing *d*–*d′* on the door jamb to *d″*, the shadow lines on the door casing may be located and *m′* found. The point *m* is the shadow of *m′* and may be found on the wall shadow line by drawing through *m′* from *A*. A vertical through *m*, parallel to the door casing, gives the shadow of its edge.

The shadow line on the ceiling of the upper edge of the shade may be found in the same way that the shadow of the table top on the floor was found. The point *a* is the projection on the ceiling of the point *A*, the source of light. The point 5″ is the projection on the ceiling of the point 5′. The line *a*–5″ continued is the intersection with the ceiling of a light plane perpendicular to the ceiling and passing through the corner 5′, and contains the light ray, *A*–5′–5′*s*. Point 5′*s* is found by drawing from *A* through 5′. We now draw a line through 5′*s* from *V*, locate 6′*s* by drawing through 6′ from *A*, draw through 6′*s* to *V′*, locate 7′*s* by a line from *A* through 7′ and draw a line from 7′*s* to *V*. Another line from 5′*s* to *V′* completes the outline.

The shadows of the corner tubes are constructed as shown at 6′*s*—by drawing from *a*, through *r* and *r′*, to *rs* and *r′s* or from *A* through the lower end of the tube to *rs* and *r′s*, and connecting *r* with *rs* and *r′* with *r′s*. In a similar way the other three shadows are drawn.

This completes the problem. Like the others, this plate should be redrawn by carefully following the instructions until the placing of every point and every line is thoroughly understood, then similar problems with variations of conditions should be solved. Like all other problems in perspective, we need to understand thoroughly only a few basic principles and we shall have no trouble in building up the others.

XIII

WHO DISCOVERED THE RULES OF PERSPECTIVE?

THE observation and study of the natural phenomena of perspective date back to ancient times; the understanding of the graphic processes of perspective as known to-day dates back about two hundred years, although some of these processes were understood a century earlier. Much earlier than this, in the scratchings and drawings on bones of the Later Paleolithic period—35,000 to 15,000 years ago, we find evidence of some cognizance of the simpler perspective phenomena. So the first observation of perspective effect is lost in the dim obscurity of a very remote past. It is as old as attempts to draw from nature.

The fundamental phenomenon of perspective is the formation of the image of an object on a real or imaginary transparent screen, the picture plane, by points where it is pierced by lines from the eye to the various points on the object when it is looked at through the screen. In fact, early investigations of perspective, such as Dürer's, were made by drawing on a sheet of glass or other transparent screen what could be seen through it. The word "perspective," meaning literally to see or look through, is derived from this phenomenon, which was recognized in the fifth century B.C.

In the introduction to Book VII of Vitruvius we find that:

> Agatharcus, at the time when Æschylus taught at Athens the rules of tragic poetry, was the first who contrived scenery, upon which subject he left a treatise. This led Democritus and Anaxagorus, who wrote thereon, to explain how the points of sight and distance ought to guide the lines, as in nature, to a centre; so that by means of pictorial deception, the real appearances of buildings appear on the scene, which, painted on a flat vertical surface, seem, nevertheless, to advance and recede.

Vitruvius is not clear as to whether the scenery itself or the treatise of Agatharcus inspired the Greek philosopher, Anaxagorus, to write of perspective, but the latter is quoted as writing that, " . . . in drawing, the lines ought to be made to correspond, according to a natural proportion, to the figure which would be traced out on an imaginary intervening plane by a pencil of rays proceeding from the eye, as a fixed point of sight, to the several points of the object viewed." Thus it is seen that the basic phenomenon of perspective was clearly understood.

As far as we know, the treatise of Agatharcus referred to by Vitruvius was the first investigation of the mathematics of perspective, and the scenes which Agatharcus painted for the tragedies of Æschylus in the fifth century before Christ were the first perspective drawings, that is, the first drawings laid out according to rules of perspective rather than merely copied from nature.

We are quite sure that these scenes were in parallel perspective and it is quite easy to see why even if anything about angular perspective were known, which is doubtful, it would not be wanted for these scenes. Being viewed from different parts of a theatre—from different points of station rather widely separated, parallel perspective offers less distortion, the planes parallel to the picture plane showing about as well from one station point as from another.

Little about the theory of angular or two-point perspective was developed in the twenty-two centuries after Agatharcus (fifth century B.C. through the seventeenth century A.D.), although several elaborate treatises on parallel or one-point perspective appeared. Many paintings of the Renaissance, however, show a keen observation of angular perspective in nature. A notable attempt to analyze the theory of angular perspective was made by Albrecht Dürer, the famous engraver, in 1525. His attempt, as well as others' of the period, was, as well illustrated in a book by a successor of Dürer's, Jan Vredeman de Vries, published in 1630, a scheme to lay out angular perspective on an elaborate network of squares drawn in parallel perspective, an elaborate scheme of plotting or craticulation. (See pages 53–55.) Jan Vredeman de Vries, however, employs an assumed horizon and gives the rule for finding vanishing points.

Parallel or one-point perspective was well understood by the Romans. The wall paintings of Pompeii (first century A.D.) were certainly not drawn directly from nature and were laid out in reasonably accurate one-point perspective. It is mystifying to think that the great painters and architects of the Renaissance in Italy did not evolve the rules of angular perspective; that the great mind of Leonardo da Vinci, who understood the phenomena of perspective well and who also realized that perspective was a mathematical as well as a graphic process, did not discover any rules for drawing in angular or two-point perspective. Leonardo had great regard for perspective. In the preamble to his discourse on the subject, he says:[1] "Perspective is the bridle and rudder of painting." Again later that:

> Perspective is a rational demonstration whereby experience confirms how all things transmit their images to the eye by pyramidal lines. By pyramidal lines I mean those which start from the extremities of the surface of bodies and by gradually converging from a distance arrive at the same point; the said point being, as I shall show, in this particular case, located in the eye, which is the universal judge of all objects.

[1] "Leonardo da Vinci's Note Books," McCurdy's translation, Chas. Scribner's Sons.

Leonardo da Vinci's notes were made early in the sixteenth century. Other masters of the Renaissance wrote on and taught the subject, but all their theories were based on one-point perspective. In 1693, Andrea Pozzo wrote a profound treatise in Latin, elaborately and beautifully illustrated, but covering the theory of parallel perspective only. Pozzo's book was translated into English by John Stuart in 1807, and even this translation is a delightful book to peruse. Some of the illustrations, parallel perspectives of the undersides of domes projected on horizontal picture planes, are really amazing.

The Renaissance brought forth many treatises on perspective by Italian, French, Dutch, and other artists, but it remained for an English *mathematician,* Dr. Brook Taylor, a follower and ardent admirer of Sir Isaac Newton, to lay down concisely, in 1715, *all* the fundamental rules upon which the science of perspective is based. Dr. Taylor's treatise in its first edition contained only 42 pages, 12mo., and 18 small plates. It may be interesting, perhaps, to quote the heading and preface of Dr. Taylor's book:

> Linear Perspective or, a New Method of Representing justly all manner of objects as they appear to the Eye in all Situations.
>
> In this Treatise I have endeavour'd to render the Art of Perspective more general, and more easy, than has yet been done. In order to do this, I find it necessary to lay aside the common Terms of Art, which have hitherto been used, such as Horizontal Line, Points of Distance Ec. and to use new ones of my own; such as

seem to be more significant of the Things they express, and more agreeable to the General Notion I have formed to my self of this Subject.

Thus much I thought necessary to say by way of Preface; because it always needs an Apology to change Terms of Art, or any way to go out of the common Road. But I shall add no more, because the shortness of the Treatise it self makes it needless to trouble the Reader with a more particular Account of it.

Dr. Taylor evidently seemed to think that he wrote a treatise on art, but his book is a collection of clean-cut mathematical theorems, brief and general, and not wasting itself over minor or easily deducible details. For this reason the book was criticized as too brief and too obscure, and it is easily conceived how dry and difficult it must have been to those who needed its lessons most,—painters and draftsmen. Soon lengthy expositions, based on Taylor's fundamentals, began to appear. Indirectly based on these fundamentals, they are still appearing, for every text-book on perspective published in nearly two centuries is a descendant of Taylor's "Essay on Linear Perspective." A most interesting elucidation of Taylor's book was published in England in 1774 by Thomas Malton. Instead of the original 42 12mo. pages, Malton's book contained 350 quarto pages and goes into minute details. Interesting indeed are the engraved illustrations, many of which have flaps so pasted that they may be folded up to make actual models of the planes involved.

From the time of Taylor and Malton to the present day, there have appeared many score treatises on perspective, in all civilized countries, in different languages, and of varying scope and value. Conspicuous among modern books are the French treatise of M. Joseph Adhémar, published in 1846, and the American book by Professor William R. Ware, first published in 1883.

INDEX

A

B

C

INDEX

I

L

M

T

V

W